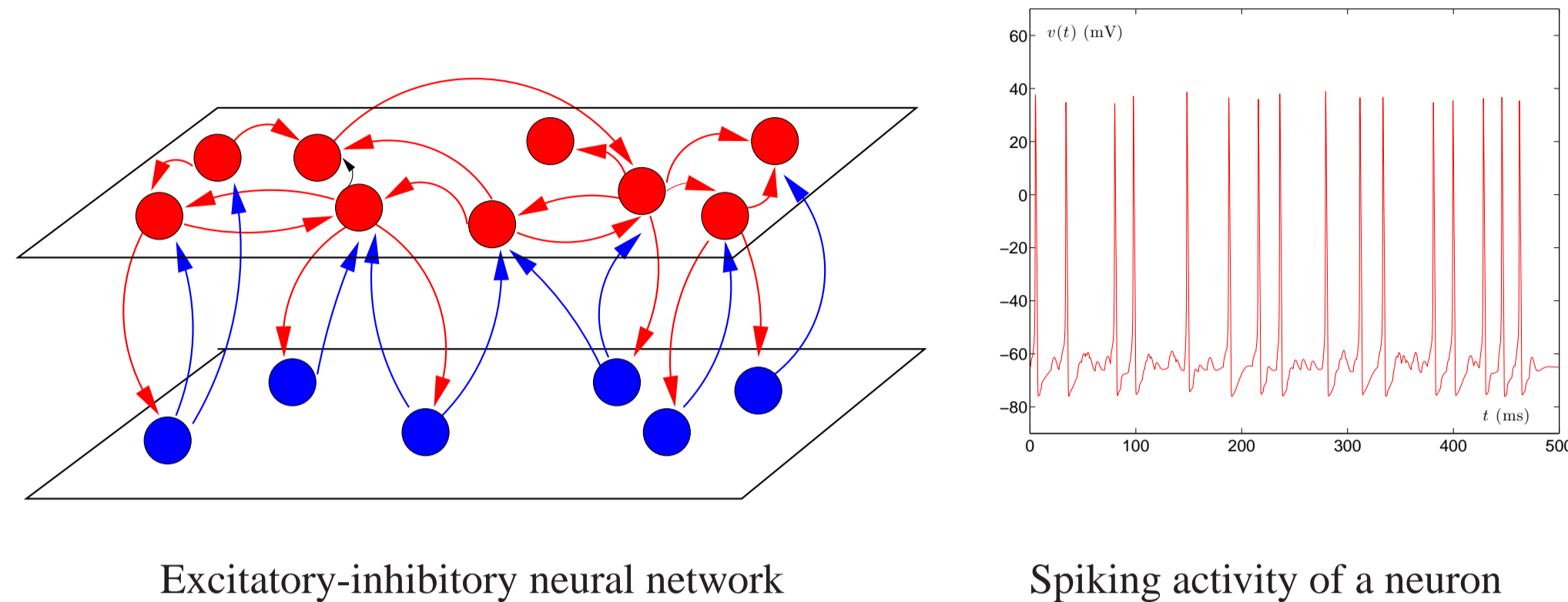


## 1 Introduction

- Computational and mathematical modeling of neural systems
- Goal: understanding of the information contents of neural signals by modeling the nervous system at different scales, from single cells to neural networks.



- Numerical schemes for the simulation of neurons and neural networks.
- Analysis of the dynamics of neurons and neural networks.

## 2 Models

**Integrate-and-fire models.** The neuron is described by his membrane potential  $v$  obeying the nonlinear differential equation

$$\frac{dv}{dt} = f(v) + I + I_{syn}(t)$$

and the reset condition

$$v(t) = \vartheta \Rightarrow v(t) = v_r \text{ and } t^f = t$$

- $I$  is an external current,  $\vartheta$  a threshold,  $v_r$  a reset potential and  $t^f$  is the so-called firing time or spike timing.
- $I_{syn}$  is the synaptic current generated by the activity of presynaptic neurons,

$$I_{syn}(t) = \sum_j w_{ij} \sum_f \alpha(t - t_j^f)$$

where  $\alpha$  is a pulse-coupling function, for instance  $\alpha(t) = \delta(t)$  (instantaneous coupling),  $\alpha(t) = \exp(-t/\tau)$  (exponential coupling) or  $\alpha(t) = g(t)(v - E_s)$  (conductance-based current) where  $g$  is a conductance and  $E_s$  a reversal potential.

**Detailed neuron models.** More details models account for numerous ion channels and include a spike-description. Reduction of these models to two dimensions can be expressed as

$$\begin{aligned} \frac{dv}{dt} &= f(v, u) + I + I_{syn}(t) \\ \frac{du}{dt} &= g(v, u) \end{aligned}$$

**Population models.** Time-dependent firing rate models are commonly described by

$$\tau \frac{dr}{dt} = -r * F(Wr + h)$$

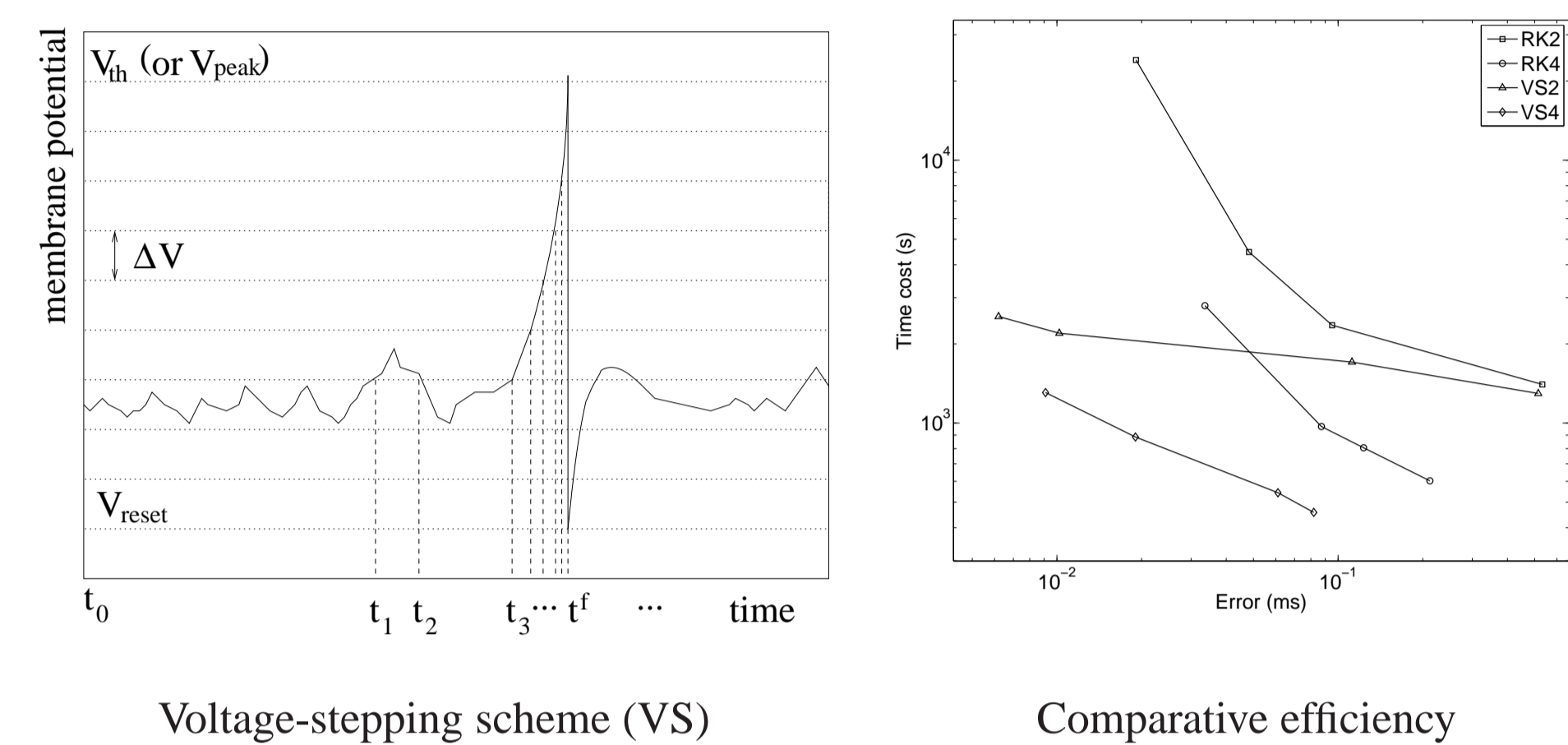
where  $r$  is the firing rate vector,  $\tau$  the relaxation time,  $W$  the matrix of the synaptic weights and  $F$  is a sigmoid-like activation function, for instance  $F(X) = H(X - \Theta)$  where  $H$  is the Heaviside function and  $\Theta$  a threshold.

## 3 Numerical Schemes

Due to discontinuities, traditional time-stepping schemes fail to compute the spike timings accurately.

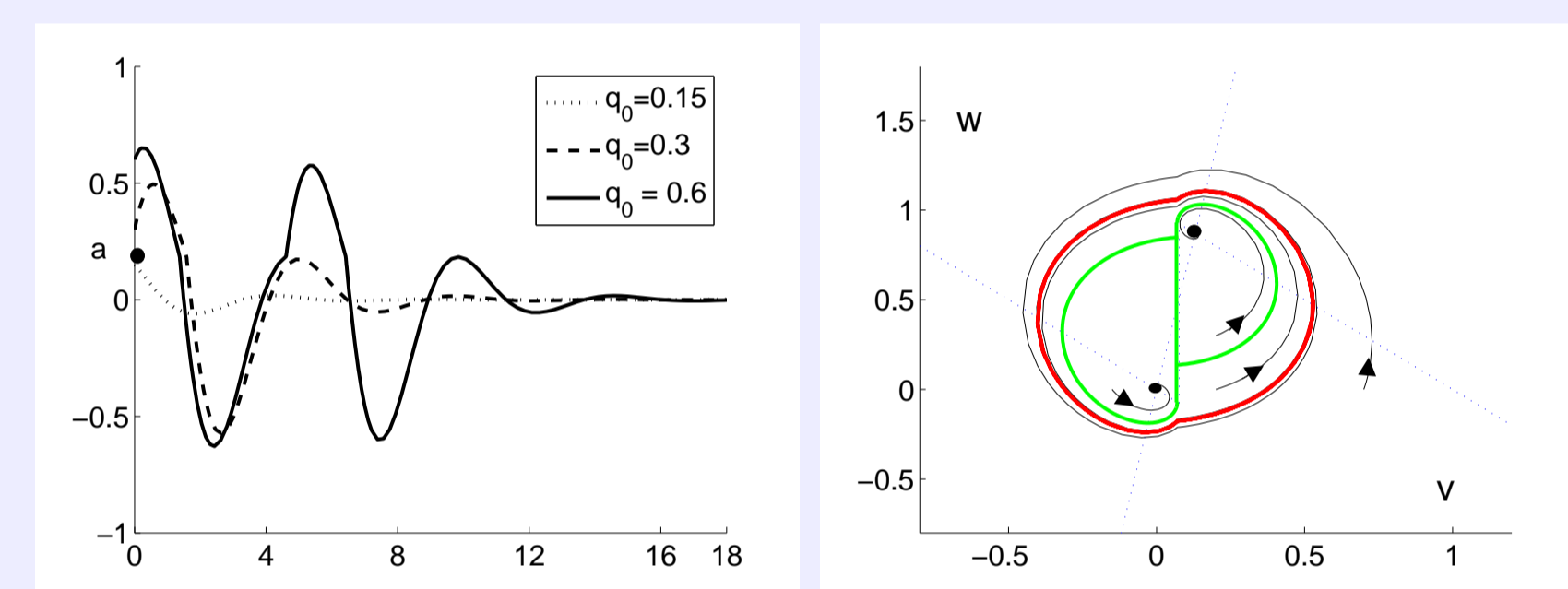
Alternative strategies have been developed :

- **Event-driven** schemes: Timings of spike are calculated exactly
- **Voltage-stepping** schemes: Implicit activity-dependent time discretization.

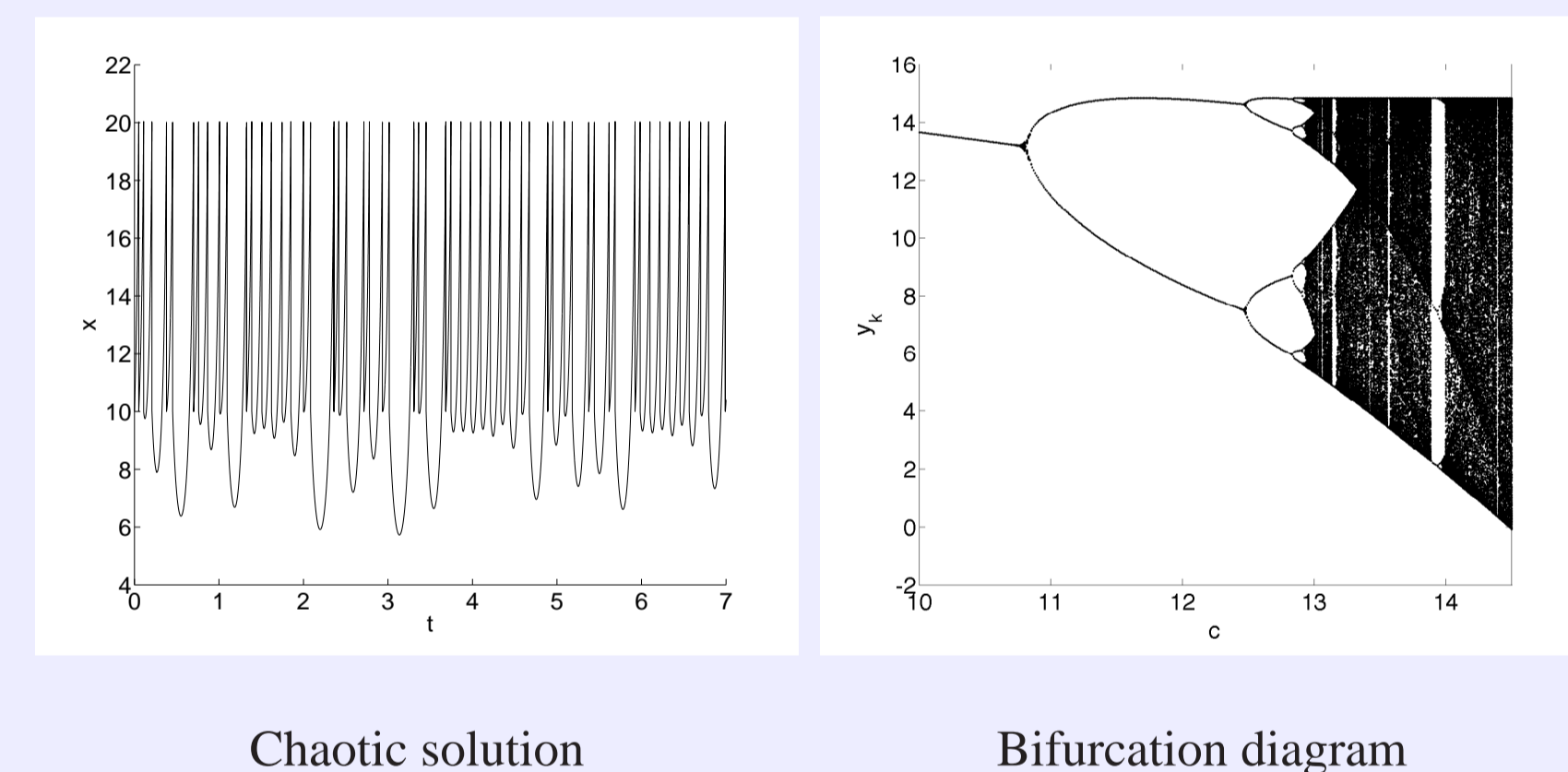


## 4 Neural Dynamics

- Excitability and oscillations - Piecewise linear approximation

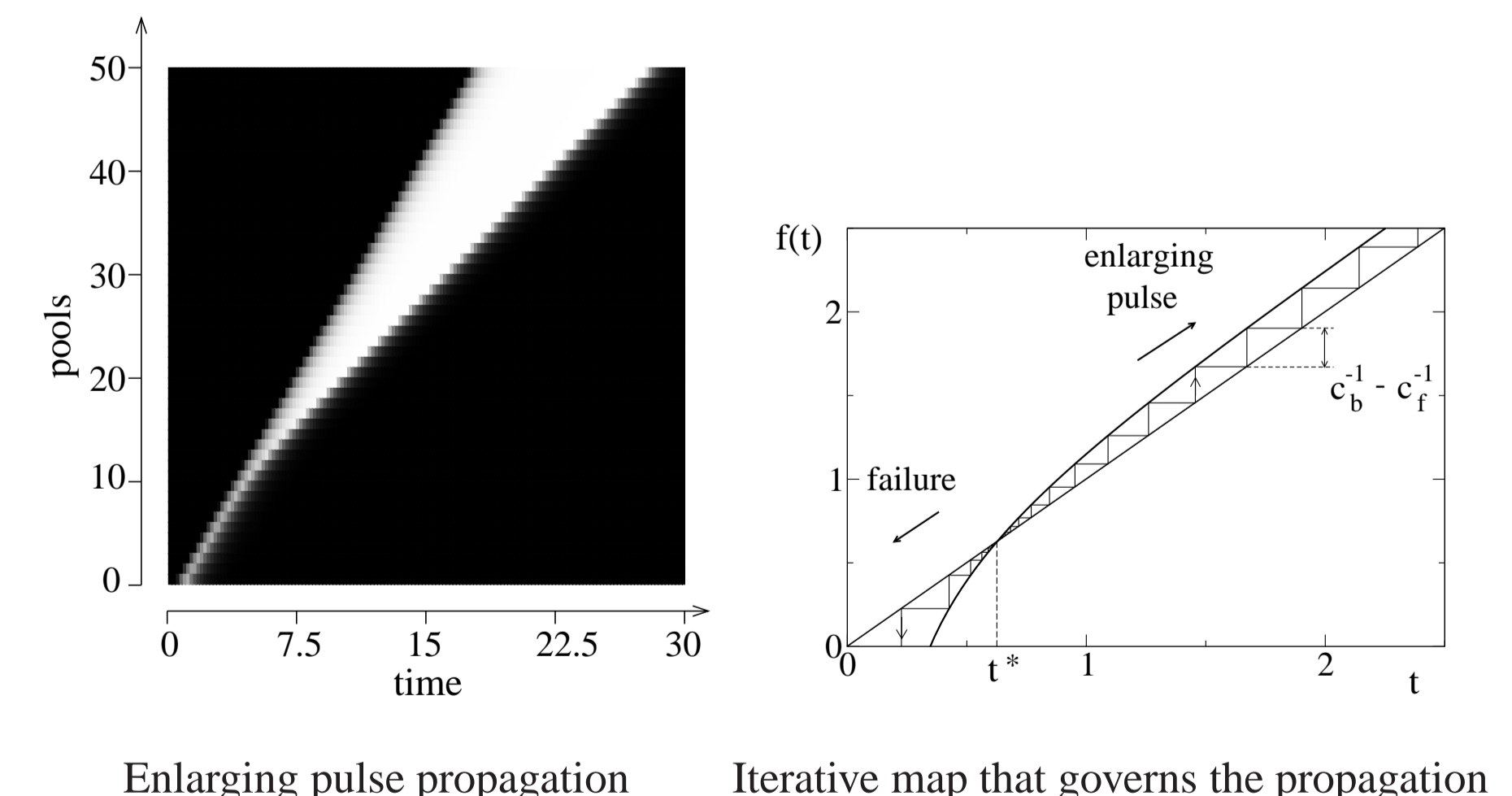


- Bifurcations and Chaos



## 5 Network Dynamics

- Synchronization
- Propagation of synaptically generated traveling waves. Propagation of pulse in an excitatory-inhibitory network.



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