Control Issues in Biped Walking

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The main particularity and the main interest about legged robots lie in the unilateral contact with the ground: since the robot is not forced to keep a continuous contact with the ground, it can travel through any uneven terrain. For example, breaking the contact with the ground might be the only way to step over an obstacle. But these unilateral constraints can not be directly controlled and therefore induce many control problems: undesired impacts or contact losses might have strong destabilizing effects on the robot's dynamics.

Working with the rigid body assumption, we introduce the lagrangian dynamics of the robot:

$$M(q)\,\ddot{q} + N(q,\dot{q}) = C(q)\,\lambda + T\,u \tag{1}$$

and a Linear Complementarity Problem (LCP) to model the contact:

$$\begin{cases} \lambda_n^T \varphi_n(q) = 0 \\ \varphi_n(q) \geq 0 \\ \lambda_n \geq 0 \\ |\lambda_t| \leq \mu_0 \lambda_n \\ \varphi_t(q) \text{ fixed (i.e.} \dot{\varphi}_t(q) = 0) \text{ when } \varphi_n(q) = 0 \end{cases} \tag{2}$$

The robot needs to push on the ground to generate any displacement: the contact is necessary for the movements of the robot. But the contact forces are mechanically bounded (2), and therefore the movements that the robot can undertake are limited. Straightforward derivations from (1) and (2) allow us to obtain the conditions on the feasible movements in the following form:

$$\begin{cases} M_1 \ddot{q} + N_1 = C_1 \lambda \\ A\lambda \geq 0 \\ C^T \ddot{q} = \ddot{\varphi}_d - s \end{cases} \tag{3}$$

Limitations on feasible movements naturally induce limitations on the stabilization properties that we will achieve. Having to deal with feedbacks which might be valid only in limited domains, the strategy we develop consists in designing a composite control law, selecting the correct feedback to apply. Suppose we have

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*INRIA BIP project, http://www.inrialpes.fr/bip, Email: Pierre-Brice.Wieber@inrialpes.fr a collection of feedbacks $u_i(x,t)$ and associated Lyapunov functions $V_i(x,t)$. We design the control law

$$u(x,t) = u_{j(x,t)}(x,t) \tag{4}$$

with

$$j(x,t) \in \operatorname*{Argmin}_{i \in \mathcal{I}} V_i(x,t) \tag{5}$$

for which the Lyapunov function

$$V(x,t) = \min_{i \in \mathcal{I}} V_i(x,t) \tag{6}$$

can be prooved to decrease. The stability of this variable structure control is assured by the particular switching law (5) which depends on the respective value of the Lyapunov functions. Employing some control laws valid in the domains $\mathcal{D}_i$, we can obtain in this way a control law valid in $\mathcal{D} = \bigcup \mathcal{D}_i$.

Now, we will consider that the switching part of the behaviour is correctly handled. Even though, we need that the building blocks $u_i(x,t)$ deal efficiently with the hybrid dynamics (1)-(2). The simplest way to achieve this is to require that the control laws keep the LCP in one given mode, which must be done in two ways:

– insuring that no active constraint is broken (insuring a steady grip of the foot on the ground), which induces a bound on the control inputs:

$$A(C^T M^{-1} C)^{-1}(C^T M^{-1} N - s - C^T M^{-1} T u) \geq 0 \tag{7}$$

– avoiding any undesired impact with an inactive constraint, which induces a bound on the states $\varphi(q) > 0$.

To take care of such inequalities on the state and on the inputs is still an open question in control theory: further research has to be done in this direction.

References

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