HYBRID DYNAMICS FOR THE SIMULATION OF REHABILITATION TO WALKING BY FES

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1 ABSTRACT

The numerical simulation of the rehabilitation to walking of disabled people by Functional Electric Stimulation can offer new tools in the design and optimization of stimulation patterns, but some Hybrid Dynamics need to be taken into account. We propose here a specific time-integration scheme for such dynamical systems, with a particular emphasis on the various measures that need to be undertaken in order to handle safely the limited numeric precision of computers.

2 INTRODUCTION

The numerical simulation of the rehabilitation to walking of disabled people by Functional Electric Stimulation can offer new tools in the design and optimization of stimulation patterns, allowing especially to test these patterns on virtual patients, avoiding therefore the different risks connected with experiments on human beings. Now, the dynamics of walking involves different phases in the contact between the feet and the ground, single support, heel contact, double support, toe off, with each time a completely different dynamical behaviour. This switching between different phases induces what is generally referred to as a *Hybrid Dynamics*, with distinct events separating qualitatively different continuous dynamics [1].

Recent models of muscle contraction specifically designed for Functional Electric Stimulation have also been introducing such Hybrid Dynamics, with distinct dynamical behaviors depending on the state of contraction of the muscle and the electric stimulation signals [2]. Precisely simulating such Hybrid Dynamics requires to handle with care the numerical errors that are bound to appear in any differential equations solver with event finding.

After presenting here the specific Hybrid Dynamics that we are considering for both the walking behaviour in section 3 and the muscle contraction in section 4, we will present in section 5 the scheme that we have designed in order to handle the different events that arise from the simulation, how they relate to the continuous

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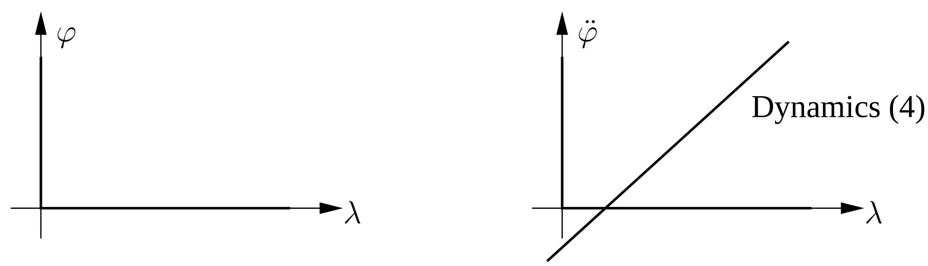

Figure 1: Signorini's complementarity law on position (left) and acceleration (right).

parts of the dynamics, and how the numerical errors are taken into account in order to lead to a simulation as precise as possible.

3 THE DYNAMICS OF WALKING

3.1 The contact between the feet and the ground

The walking behaviour takes its origin in the contact between the feet and the ground, a contact which can be represented mathematically with the help of Signorini's complementarity law [3] shown on the left of Figure 1: when the height $\varphi(q)$ of a foot above the ground is greater than zero (the vector q will be considered to describe the geometric configuration of the person), the contact force λ can only be zero, and when this height is zero, when the foot is in contact with the ground, this force can take positive values and only positive values, what corresponds to the fact that the feet can push on the ground but they can't pull on it. This relationship can be expressed as well in the following way,

$$\varphi(q) \geq 0, \tag{1}$$

$$\lambda \geq 0, \tag{2}$$

$$\lambda^T \varphi(q) = 0. \tag{3}$$

We can observe this way that depending on the state of the contact between the feet and the ground the forces acting on the person will be qualitatively completely different.

3.2 The acceleration of the person

When the feet are in contact with the ground, their vertical acceleration $\ddot{\varphi}$ can only be positive, going upwards, and we can consider a second Signorini law relating this acceleration with the contact forces as shown on the right of Figure 1: when

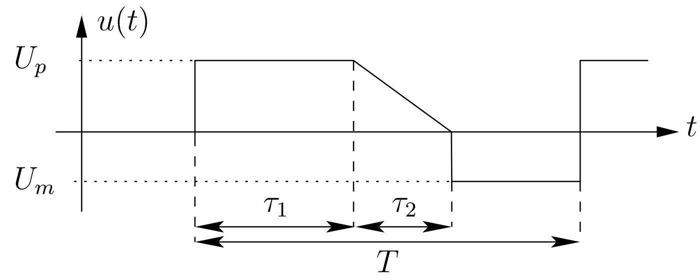

Figure 2: The calcium concentration in the muscles.

this acceleration is greater than zero, the contact is detached and the contact force λ can only be zero. Now, the dynamics of mechanical systems implies a linear relation between forces and accelerations, so depending on where this linear law intersects Signorini's graph, the contact will detach with a zero contact force, or will remain active with a positive force (Figure 1). These two possibilities will once again imply qualitatively very different behaviors. This relationship can be expressed in the following way,

$$M(q)\,\ddot{q} + N(q,\dot{q}) \;=\; \gamma + C(q)^T\lambda, \tag{4}$$

$$\ddot{\varphi} = C(q)\,\ddot{q} + s(q,\dot{q}) \;\geq\; 0, \tag{5}$$

$$\lambda \;\geq\; 0, \tag{6}$$

$$\lambda^T\ddot{\varphi} \;=\; 0, \tag{7}$$

where $M(q)$ is the inertia matrix of the system, $N(q,\dot{q})$ the inertial nonlinear forces (gravity, centrifugal and Coriolis forces), γ the action of the muscles and $C(q)$ the Jacobian of the position of the contact points.

4 THE DYNAMICS OF FUNCTIONAL ELECTRIC STIMULATION

4.1 Dynamics of the calcium concentration in the muscles

In the model of muscle contraction specifically designed in [2] for FES applications, each electric impulse sent to the muscles is considered to induce first of all the following variation of the calcium ion concentration $u(t)$ in the muscles: after a delay of τ seconds, this concentration rises to a positive constant U_p for τ_1 seconds, then decreases for τ_2 seconds before falling to a negative constant U_m (Figure 2). This whole process is re-initialized when the next impulse is sent after T seconds, very similar to a clockwork: this will be referred to as the *calcium clock*.

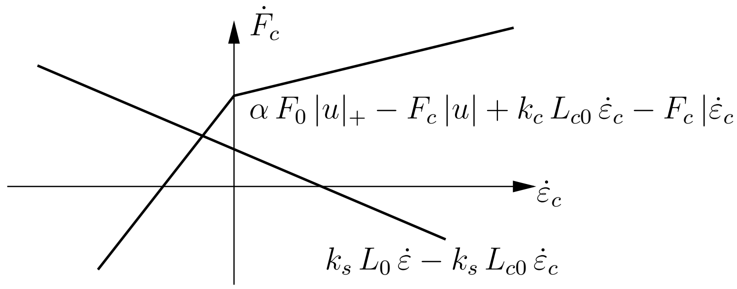

Figure 3: The dynamics of the contractile elements of the muscles.

4.2 Dynamics of the contractile elements of the muscles

Now, the dynamics of the contractile element of a muscle considered to be coupled in series with a linear spring can be written as

$$\dot{k}_c = -(|u| + |\dot{\varepsilon}_c|)\, k_c + \alpha\, k_0\, |u|_+, \tag{8}$$

$$\dot{F}_c = -(|u| + |\dot{\varepsilon}_c|)\, F_c + \alpha\, F_0\, |u|_+ + k_c\, L_{c0}\, \dot{\varepsilon}_c, \tag{9}$$

$$\dot{F}_c = k_s\, (L_0\, \dot{\varepsilon} - L_{c0}\, \dot{\varepsilon}_c), \tag{10}$$

with $\dot{k}_c$ the stiffness of the contractile element, $\dot{F}_c$ the force it generates and $\dot{\varepsilon}_c$ its variation of length [2]. The two last equations describe a line and two half-lines which need to be intersected as shown in Figure 3. Depending on where they intersect (verifying beforehand that they do intersect at a unique point), the muscle will contract or not, implying once again qualitatively very different behaviors.

5 NUMERICAL INTEGRATION OF HYBRID DYNAMICS

5.1 General time-integration scheme

We can observe through Figures 1, 2 and 3 that when the state of the contacts between the feet and the ground, the state of the calcium clocks or the state of the contractile elements of the muscles changes, the dynamics of the walking behaviour under the action of Functional Electric Stimulation strongly changes qualitatively and not only quantitatively. When numerically integrating this dynamics with time, it is necessary therefore to detect these changes and take them into account properly for the sake of both numerical precision and computation speed.

Note that such instantaneous changes between qualitatively different dynamical behaviors give rise to what is generally referred to as a Hybrid Dynamics. A general time-integration scheme for such Hybrid Dynamics can be devised in the following way, shown in Figure 4: depending on the current state of the contacts,

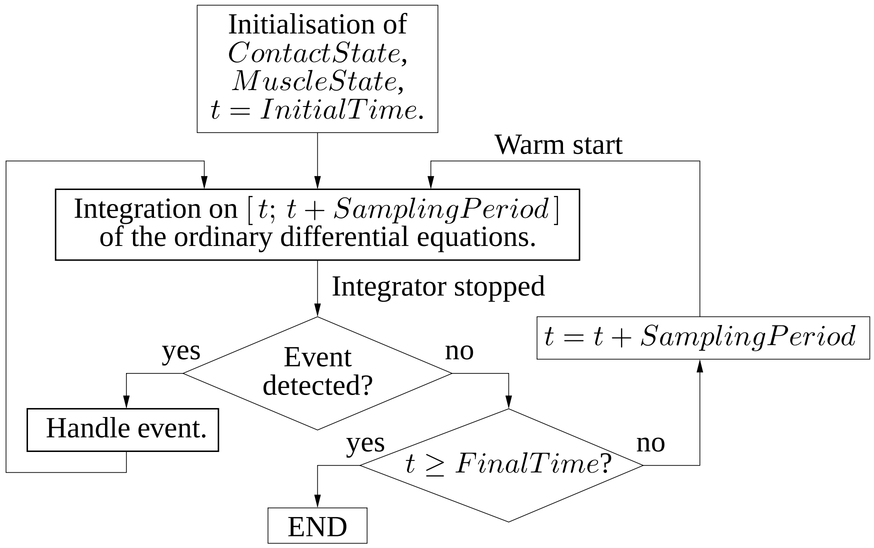

Figure 4: General time-integration scheme.

calcium clocks and contractile elements, integrate numerically the corresponding set of Ordinary Differential Equations. If an event such as a change of state is detected by the numerical integration scheme (following the method described in the next section), then the time-integration is stopped, proper handling of the event is undertaken, and the time-integration is restarted, taking care of the new state of the contacts, calcium clocks and contractile elements.

5.2 Detecting events and handling the numerical precision

General numerical ODE integration schemes usually propose event detection facilities based on detecting when some functions $f_k(q, \dot{q}, \ldots)$ cross zero: all the different events that we need to detect must be turned therefore in this form, in our case all the changes in the states of the contacts, calcium clocks and contractile elements. This is of no particular theoretical difficulty in our case, but this unfortunately raises many problems related with numeric precision which need proper treatment.

First of all, comparisons with zero can't be trusted from the point of view of computer algebra, so "approximations" of zero need to be considered, most usually the largest number ε_n such that $1 + \varepsilon_n = 1$ (with the limited numeric precision of computer algebra), and its square root $\sqrt{\varepsilon_n}$. On modern computers, we usually have $\varepsilon_n = 2^{-52} = 2.22 \cdot 10^{-16}$ and therefore $\sqrt{\varepsilon_n} \approx 1.49 \cdot 10^{-8}$, both of these values being negligible when taken as meters or seconds.

Since comparisons with zero can't be trusted, some hysteresis must be introduced in the event detection, detecting when the functions $f_k(q, \dot{q}, \ldots)$ cross zero in one

way, and a value slightly different from zero in the other way: fortunately, we've seen that even a hysteresis of $\sqrt{\varepsilon_n}$ can be considered to be globally negligible.
Moreover, the choice of the functions $f_k(q, \dot{q}, \ldots)$ must be carefully decided in order to avoid the risk of inconsistent states. Specifically in the case of Signorini's complementarity law (Figure 1), detecting when $\varphi(q)$ crosses zero in one way and when λ crosses zero in the other way can be risky and it can be preferred to check only the variations of one of these two functions, for example $\varphi(q)$.
Finally, the event detection schemes of general ODE solvers can be tricked and miss some events when they appear too close in time one from the other, of the order of ε_n seconds. This appears to be particularly risky in the case of the calcium clocks which can be explicitly synchronized by the external electric impulses: they need therefore to be all shifted one from the other by at least $100\,\varepsilon_n$ seconds to avoid any risk.

6 CONCLUSION

The walking behaviour under the action of Functional Electric Stimulation presents a wide variety of qualitatively very different dynamical behaviors, depending on the state of the contacts between the feet and the ground, the state of the calcium clocks and the state of the contractile elements of the muscles. In order to realize a precise and efficient simulation of such a Hybrid Dynamics, we propose here a specific time-integration scheme, with a particular emphasis on the various measures that need to be undertaken in order to handle safely the limited numeric precision of computer algebra. This scheme has been implemented successfully in the HuMAnS toolbox [4] (for Humanoid Motion Analysis and Simulation), a Scilab [5] toolbox distributed in Open Source for the simulation of rehabilitation to walking by FES.

References

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