A tracking control scheme for complementarity Lagrangian systems

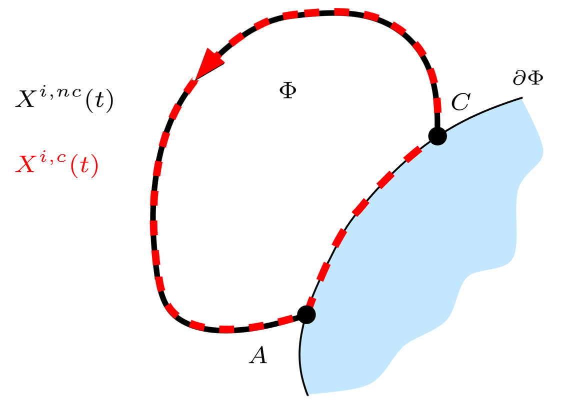
Jean-Matthieu Bourgeot, Bernard Brogliato

Jean-Matthieu.Bourgeot@inrialpes.fr, INRIA Rhône-Alpes, 655 avenue de l'Europe - 38330 Montbonnot Saint-Martin - France

équipe BIPOP team

Objective

The focus of this work is the tracking control of a class of nonsmooth fully actuated Lagrangian systems. These systems we deal with in this work, may evolve in three different phases of motion : i) a **free motion** phase, ii) a **permanently constraint** phase and iii) a **transition** phase whose goal is to stabilise the system on some surface $\partial\Phi$. During the transition phase the system is subject to unilateral constraints, and collision occur. The aim of this work is to study a control scheme which guarantees some stability properties of the closed loop system during such motions.



$$\Omega_{2k} \xrightarrow[I_k]{} \Omega_{2k+1} \xrightarrow[\text{LCP}(\lambda)]{} \Omega_{2k+2}$$

In the time domain one gets a representation as :

$$I\!R^+ = \underbrace{\Omega_0 \cup I_0 \cup \Omega_1}_{\text{cycle 0}} \cup \Omega_2 \cup I_1 \cup \ldots \cup \Omega_{2k-1} \cup \underbrace{\Omega_{2k} \cup I_k \cup \Omega_{2k+1}}_{\text{cycle k}} \cup \ldots \quad (1)$$

where Ω_{2k} denotes the time intervals associated to free-motion phases and Ω_{2k+1} those for constrained-motion phases. The transition $\Omega_{2k+1} \longrightarrow \Omega_{2k+2}$, does not define a specific phase (or DES mode) because it does not give rise to a new type of dynamical system.

Dynamic model

The dynamics of the system may be written as:

$$M(X)\ddot{X} + C(X,\dot{X})\dot{X} + G(X) = u + \nabla F(X).\lambda_X$$
$$F(X) \geq 0 \;,\; F(X)^T \lambda_X = 0 \;,\; \lambda_X \geq 0$$

Impact model

A collision rule is needed to integrate this system and to render the set Φ invariant. In this work, it is chosen as in [3]:

$$\dot{X}^+ = -e_n\dot{X}^- + (1+e_n) \arg\min_{z\in T_\Phi} \frac{1}{2}[z-\dot{X}^-]^T M(X)[z-\dot{X}^-]$$

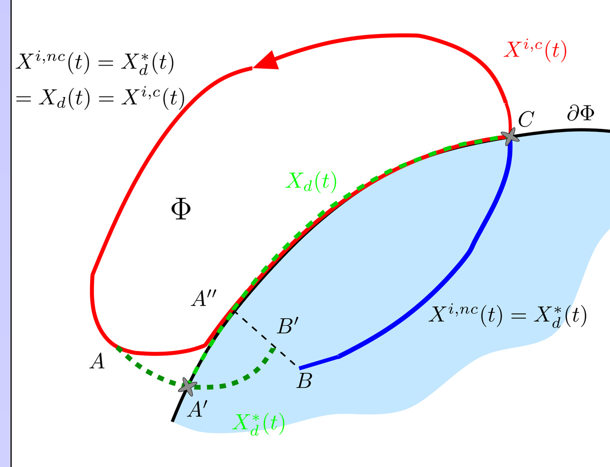
where $\dot{X}^+$ is the post impact velocity, $\dot{X}^-$ is the pre-impact velocity, T_Φ the tangent cone to the set Φ at $X(t)$ and e_n is the restitution coefficient, $e_n \in [0,1]$.

Why not tangential approach ?

In the objectives, we have seen that impacts occur during the transition phase, and these impact may disturb the stability of the closed-loop system. A solution could be landing on the surface $\partial\Phi$ without normal velocity. And then no impact occur : It is the tangential approach. But

- Even if the desired trajectories are impactless, due to non-zero initial tracking errors, impacts may occur. Then, in any case, collisions have to be incorporated into the stability analysis
- This is not a robust strategy since a bad estimation of the constraint position, may result to no stabilisation at all on $\partial\Phi$. Consequently it is a much better strategy to impose collisions for stabilisation on $\partial\Phi$.
- The good strategy for stabilisation on $\partial\Phi$ is to impose closed-loop dynamics which mimics the bouncing-ball dynamics $\ddot{X} = -g$, $X \geq 0$. This is very robust with respect to the constraint position uncertainties.

Cyclic tasks



The desired trajectory $CAA'B'BC$ tends to penetrate the surface for imposing stabilisation on $\partial\Phi$. But if the desired trajectory always violate the constraint then the impacts occur always and the tracking can not be asymptotically stable. The strategy is to use a desired trajectory which evolve during the cycle. The curve AA'B is adapted in function of the tracking error. It will tend to the tangential approach $CAA'BC$ when the tracking is perfect. The jump $B'B$ corresponds to the application of the force control.

Stability framework

Definition 1 (Ω**-weakly stable system)** *The closed-loop system is Ω-weakly stable if for each $\epsilon > 0$, there exists $\delta(\epsilon) > 0$ such that $\| x(0) \| \leq \delta(\epsilon) \Rightarrow \| x(t) \| \leq \epsilon$ for all $t \geq 0$, $t \in \Omega = \cup_{k\geq 0}\Omega_k$. Asymptotic weak stability holds if in addition $x(t) \longrightarrow 0$ as $t \longrightarrow +\infty$, $t \in \Omega$. Practical Ω-weak stability holds if there is a ball centered at $x = 0$, with radius $R > 0$, and such that $x(t) \in B(0,R)$ for all $t \geq T$; $T < +\infty$, $t \in \Omega$, $R < +\infty$.* ▪

Definition 2 (Strongly stable system) *The system is said strongly stable if: (i) it is Ω-weakly stable, (ii) on phases I_k, P_{Σ_I} is Lyapunov stable with Lyapunov function V_{Σ_I}, and (iii) the sequence $\{t_k\}_{k\in\mathbb{N}}$ has a finite accumulation point $t_\infty < +\infty$.* ▪

Claim 1 (Ω**-Weak Stability [1])** *Assume that the task is as in (1), and that*

(a)- *$\lambda[\Omega] = +\infty$,*
(b)- *for each $k \in \mathbb{N}$, $\lambda[I_k] < +\infty$,*
(c)- *$V(x(t_f^k), t_f^k) \leq V(x(\tau_0^k), \tau_0^k)$,*
(d)- *$V(x(.),.)$ uniformly bounded on each I_k.*

If on Ω, $\dot{V}(x(t),t) \leq 0$ and $\sigma_V(t_k) \leq 0$ for all $k \geq 0$, then the closed-loop system is Ω-weakly stable. If $\dot{V}(x(t),t) \leq -\gamma(\| X \|)$, $\gamma(0) = 0$, $\gamma(\cdot)$ strictly increasing, then the system is asymptotically Ω-weakly stable. ▪

Claim 2 (Strong Stability) *The system is strongly stable if in addition to the conditions in claim 1 one has:*

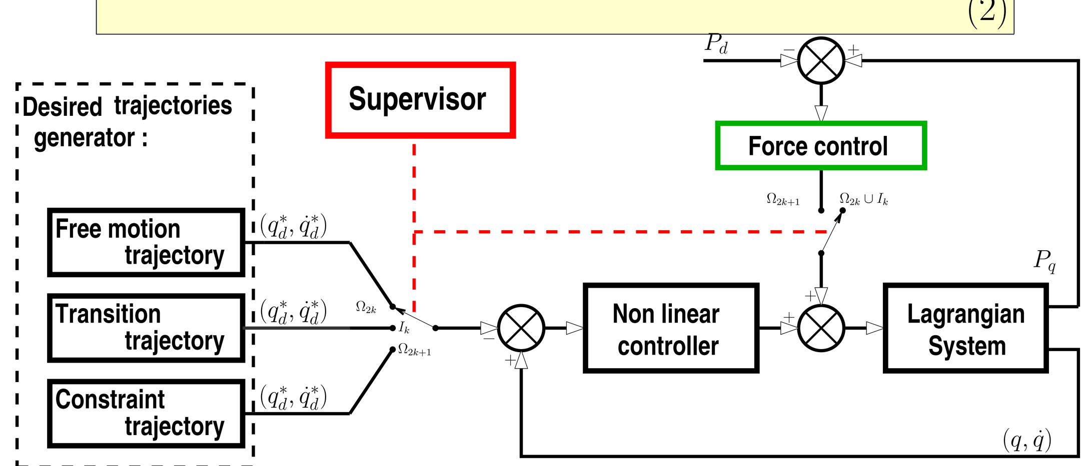
- *$V(t_{k+1}^-) \leq V(t_k^-)$;*
- *V is uniformly bounded and time continuous on $I_k - \cup_k\{t_k\}$.*

Then the system is strongly stable in the sense of definition 2. ▪

Controller structure

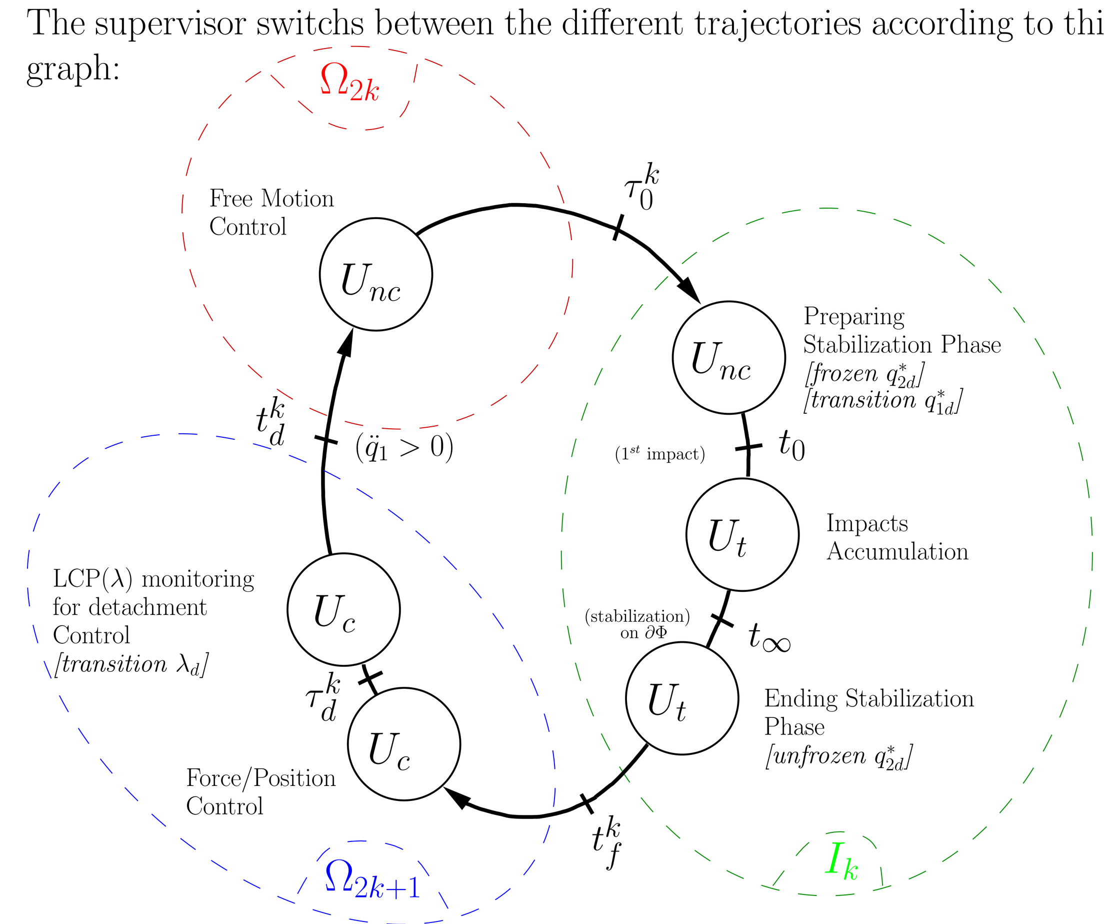
According with the transformation show in [2], q_1 is the constraint coordinate, and q_2 the tangential ones. The control laws are define as:

$$T(q)u = \begin{cases} U_{nc} = M(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + g(q) - \gamma_1 s \\ U_t \;\;= U_{nc} \quad \text{(before the first impact)} \\ U_t \;\;= M(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + g(q) - \gamma_1 \bar{s} \;\text{(after)} \\ U_c \;= U_{nc} - P_d + K_f(P_q - P_d) \end{cases} \quad (2)$$



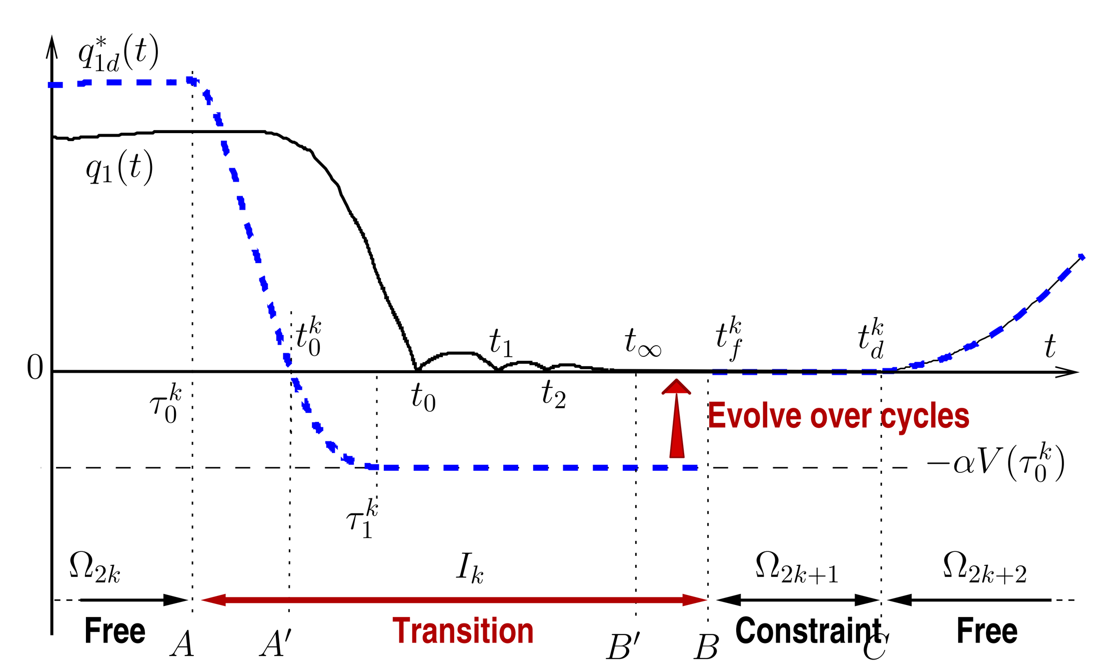
Supervisor

The supervisor switchs between the different trajectories according to this graph:



Desired trajectories on transition phases

The desired trajectories are depicted on this figure. During the transition phase the tangential reference trajectory q_{2d}^* is frozen. And the constraint desired trajectory decreases toward $-\alpha V(\tau_0^k)$.

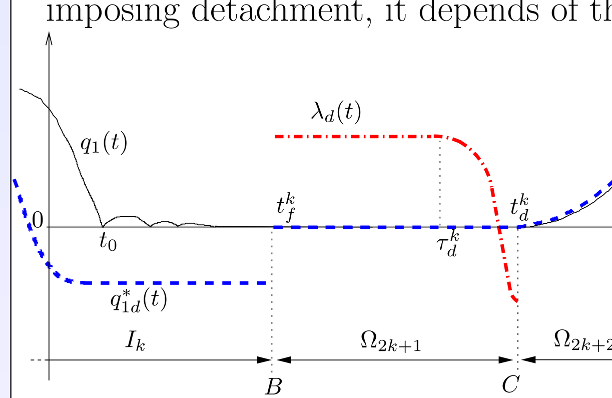


Conditions for take off

Detachment if $\ddot{q}_1(t_d^k) > 0$

$$\ddot{q}_1(t_d^k) = \ddot{q}_{1d}(t_d^k) - K_1(q)\lambda_d - \left[K_2(q)\tilde{q}_2(t_d^k) + K_3(q)\dot{\tilde{q}}_2(t_d^k)\right]$$

Due to the **coupling**, $\lambda_d = 0$ and $\ddot{q}_{1d}^*$ are not necessary sufficent for imposing detachment, it depends of the sign of $\tilde{q}_2$ and $\dot{\tilde{q}}_2$.



The strategy is to decrease λ_d until the term $K_1(q)\lambda_d$ compensate for the coupling effect, even if imposing a negative λ_d is senseless (according to the unilateral condition). The end of the decreasing of λ_d is determining by monitoring $\ddot{q}_{1d}(t)$: t_d^k is a state base event.

Results

Due to the collision rule used, there is a loss of kinetic energy at each impact time. The desired trajectories are chosen such as the function $V(t)$ decreases at each impact (except at the first one, because of the uncertainty of this first impact time). Then the stability of the closed-loop system depends of the sign of the first jump of $V(t)$ (noted $\sigma_V(t)$) for each cycle.

Claim 3 (Ω**-Weak Stability)** *Let us assume that (a) and (b) in claim (1) hold, and that*

(a)- *outside phases I_k one has $\dot{V}(t) \leq -\gamma V(t)$ for some $\gamma > 0$,*
(b)- *inside phases I_k one has $V(t_{k+1}^-) - V(t_k^+) \leq 0$, for all $k \geq 0$,*
(c)- *the system is initialized on Ω_0 with $V(\tau_0^0) \leq 1$,*
(d)- *$\sum_{k\geq 0}\sigma_V(t_k) \leq KV^\kappa(\tau_0^k) + \epsilon$ for some $\kappa \geq 0$, $K \geq 0$ and $\epsilon \geq 0$.*

Then there exists a constant $N < +\infty$ such that $\lambda[t_\infty^k, t_f^k] = N$, for all $k \geq 0$ (the cycle index), and such that:

(i)- *If $\kappa \geq 1$, $\epsilon = 0$ and $N = \frac{1}{\gamma}\ln(\frac{1+K}{\delta})$ for some $0 < \delta < 1$, then $V(\tau_0^{k+1}) \leq \delta V(\tau_0^k)$. The system is asymptotically weakly stable.*
(ii)- *If $\kappa < 1$, then $V(\tau_0^k) \leq \delta(\gamma)$, where $\delta(\gamma)$ is a function which can be made arbitrarily small by increasing γ. The system is practically Ω-weakly stable with $R = \alpha^{-1}(\delta(\gamma))$.* ▪

Conclusions

This work deals with the tracking control of fully actuated Lagrangian systems subject to frictionless unilateral constraints. These dynamical systems are named *complementarity systems* because they involve complementarity conditions. They are *nonsmooth* because the velocity may possess discontinuities (at impact times), so that the acceleration and the contact force are measures. They may be seen as a complex mixture of ordinary differential equations, differential-algebraic equations, and measure differential equations. The extension of the tracking control of unconstrained (or persistently constrained) Lagrangian systems, towards complementarity Lagrangian systems, is not trivial. The aim of this work is to study the design of a feedback controller for these specific nonsmooth systems, supposed to perform a general cyclic impacting task. First the stability framework dedicated to study these systems is recalled, and some definitions and claims are given. Then we focus on the condition of existence of closed-loop trajectories (usually called desired trajectories in unconstrained motion tracking control) which assure the asymptotic stability in closed-loop, i.e. the asymptotic convergence of the generalized coordinates towards some closed-loop invariant trajectory.

References

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[2] H.P. Huang and N.H. McClamroch. Time optimal control for a robotic contour following problem. *IEEE J. Robotics and Automation*, 4:140–149, 1988.

[3] J.J. Moreau. Unilateral contact and dry friction in finite freedom dynamics. In *Nonsmooth Mechanics and Applications*, CISM Courses and Lectures no 302. Springer-Verlag, 1988.