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# ADAPTIVE MOTION CONTROL OF ROBOT MANIPULATORS: A UNIFIED APPROACH BASED ON PASSIVITY

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## SUMMARY

This paper presents a unified approach to direct adaptive motion control laws for robot manipulat that have been studied during the last few years by several authors. It provides a general approach bas on passivity to demonstrate the global asymptotic stability of adaptive schemes applied to ri multilinked manipulators. It is shown that most of the schemes fit within this framework, which prese the advantage of being more systematic than other techniques and therefore will enable a unifi presentation of the several schemes proposed to date and will increase our understanding of adapt control of robot manipulators.

## 1. INTRODUCTION

The last few years have witnessed an increasing interest in adaptive control of rob manipulators and several different schemes have been proposed. The first robot adapti control algorithms were based on local linearization techniques.[1] The main drawback of th approach is the lack of global stability. More recently adaptive schemes based on line parametrization of the robot dynamics have emerged.[2–14] We will focus our attention on the latter, which can in turn be separated into two classes: (i) inverse dynamics based contr schemes assuming joints acceleration measurements or inversion of the estimated inert matrix,[2–5] and (ii) schemes which use only joint velocity and position measurements a which do not require any inversion of the estimated inertia matrix.[6–14] In view of the practic limitations of the earlier ones, we will consider here just those schemes using only joi position and velocity measurements. Passivity will be used as an analysis tool as advocat in Reference 15, where the authors considered the analysis of References 6 and 7 from t hyperstability point of view.

This paper exploits the passivity properties of mechanical manipulators to present adaptive control scheme based on passivity. It provides a general framework that most of t

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schemes proposed to date can be turned into. The simplicity of the approach will increase our understanding of direct adaptive control of robot manipulators.

The paper is organized as follows. Section 2 presents the notation and robots dynamics. A unified equivalent scheme of robot adaptive control is presented in Section 3. Section 4 is devoted to showing that almost all direct adaptive laws can be turned into that scheme. In order to complete the proof of stability, we show in section 5 that all the signals within the system remain bounded, utilizing passivity results previously stated. Conclusions are given in Section 6 and most of the proofs are given in the Appendices A–D.

## 2. DYNAMIC MODEL OF ROBOT MANIPULATORS

Consider a rigid, $n$-degree-of-freedom manipulator. Using the Lagrangian formulation, one can show that the dynamic model of such a mechanical system can be written as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{1}$$

Where: $q, \dot{q}, \ddot{q}$ represent the joint position, velocity and acceleration $n \times 1$ vectors, $M(q)$ is the $n \times n$ inertia matrix, $C(q,\dot{q})$ is a $n \times n$ matrix which represents centripets and Coriolis inertial terms, $g(q)$ represents the gravity torque $n \times 1$ vector and $\tau$ is the $n \times 1$ torque vector supplied by the joint actuators.

Though it is obvious that robot manipulators are complex nonlinear systems, the following properties make them appear as a particular class of these systems, and facilitate their study and design (see References 17 and 18 for details):

(1) $M(q)$ is a symmetric positive-definite matrix, lower-bounded for $q \in R^n$.
(2) Consider all the parameters of interest in adaptive control such as link masses, moments of inertia etc., represented by an $n \times r$ vector $\theta$. Then the robot dynamic equations can be expressed as a linear relationship between these unknown parameters and torque $\tau$:

$$\tau = Y(q,\dot{q},\ddot{q})\theta$$

where $Y(q,\dot{q},\ddot{q})$ is a $n \times r$ matrix of known functions, known as the regressor.
(3) There exists a particular definition of $C(q,\dot{q})$ such that the matrix

$$N(q,\dot{q}) = C(q,\dot{q}) - \frac{1}{2}\dot{M}(q)$$

is skew-symmetric.

## 3. PASSIVITY RESULT

As will be shown in the next section, it is always possible (at least for the class of control laws considered here) to interpret the closed-loop error equation obtained by replacing the control law into the manipulator dynamics as a feedback system composed of three main blocks (Figure 1) where:

(1) B1 contains signals resulting from the closed-loop equation and is always strictly passive.
(2) B2 contains the robot-dynamics (with $g(q) = 0$), i.e.

$$\bar{\tau} = -[M(q)\dot{v} + C(q,\dot{q})v] \tag{2}$$

(3) B3 contains the parameter adaptation algorithm.



(4) In order to avoid acceleration measurement, $v$ is obtained by filtering the tracking err

$$v = H(s)^{-1}\tilde{q}$$

where the transfer function $H(s)$ will be specified later and the tracking error is given

$$\tilde{q}(t) = q(t) - q_d(t)$$

$q_d$ denoting the twice differentiable bounded desired trajectory.

We are now able to apply the following theorem (Reference 16, p. 181).

*Theorem*

Consider the scheme in Figure 1. Assume B1 is a strictly passive system. If B2 and B3 passive blocks, then

$$v(t) \in L_2^n$$

*Proof.* The proof follows from standard passivity arguments (see References 15 and 16). order to prove that the tracking error signal converges to zero, we now invoke the follow lemma (Reference 16, p. 59).

*Lemma*

Let

$$\tilde{q} = H(s)v$$

where $H(s)$ is an $n \times m$ strictly proper, exponentially stable transfer function.
Then $v \in L_2^m$ implies that $\tilde{q} \in L_2^n \cap L_\infty^n$, $\dot{\tilde{q}} \in L_2^n$, $\tilde{q}$ is continuous and $\tilde{q} \to 0$ as $t \to \infty$.

## 4. SYSTEM EQUIVALENT REPRESENTATION OF SEVERAL ADAPTIVE SCHEM

We present in this section several adaptive control laws that can be studied using the passi result presented in the previous section. For each of them, we will demonstrate that:

(1) they can be analysed as an interconnection of the three main subsystems in Figure
(2) the blocks B1 and B3 are strictly passive and passive respectively.

First, we show that the subsystem B2 which contains the robot dynamics is passive.

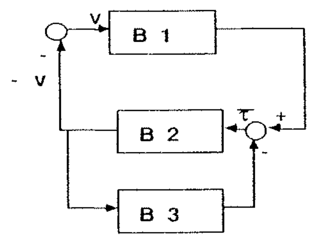


Figure 1. Closed-loop equivalent system

*Passivity of the robot dynamics B2 block.* We have

$$\bar{\tau} = -[M(q)\dot{v} + C(q,\dot{q})v]$$

Therefore

$$\langle \bar{\tau}, -v\rangle_T = \int_0^T v^{\mathrm{T}}\bar{\tau}\,dt = \int_0^t (v^{\mathrm{T}}M(q)\dot{v} + v^{\mathrm{T}}C(q,\dot{q})v)\,dt$$

$$= \int_0^T \left\{\frac{1}{2}\frac{d}{dt}(v^{\mathrm{T}}M(q)v) - \frac{1}{2}v^{\mathrm{T}}\dot{M}(q)v + v^{\mathrm{T}}C(q,\dot{q})v\right\}dt$$

$$= \frac{1}{2}\int_0^T \frac{d}{dt}(v^{\mathrm{T}}M(q)v)\,dt + \int_0^T v^{\mathrm{T}}\left(C(q,q) - \frac{1}{2}\dot{M}(q)\right)v\,dt$$

Knowing that $C(q,\dot{q}) - \frac{1}{2}\dot{M}(q)$ can be chosen as a skew-symmetric matrix (see, for example, Reference 17), we obtain

$$\langle \bar{\tau}, -v\rangle_T \geqslant -\frac{1}{2}v^{\mathrm{T}}(0)M(q(0))v(0)$$

The remainder of the section will be organized as follows.

(1) First we show that the adaptive laws proposed in References 6–8 can be treated together, in the sense that only the block B1 changes from one to another.
(2) Then we perform a separate analysis for each law.[9-14]

*System equivalent representation of schemes proposed in References 6–8*

We now consider the first three laws proposed in References 6–8. For each case the control law is given by

$$\tau = \hat{M}(q)\dot{a} + \hat{C}(q,\dot{q})a + \hat{g}(q) - H(v,\tilde{q}) \tag{6}$$

with $a = \dot{q}_d - \lambda\tilde{q}$, $v = \dot{\tilde{q}}$, $\lambda > 0$ where $a$ is a signal resulting from an outloop control law: $a = a(q,\dot{q},q_d,\dot{q}_d)$ and $\hat{(.)}$ represents the estimates of (.) at time $t$ and $\tilde{(.)} = \hat{(.)} - (.)$. $H$ is an operator to be defined later (see Reference 18 for details).

Substituting this control law into the robot dynamics, one gets the following error equation:

$$M(q)\dot{v} + C(q,\dot{q})v + H(v,\tilde{q}) = Y(q,\dot{q},\dot{a},a)\tilde{\theta} \tag{7}$$

where the three blocks B2, B1, and B3 clearly appear. The update law is given by:

$$\dot{\hat{\theta}} = -\Gamma Y^{\mathrm{T}}v \qquad \Gamma = \Gamma^{\mathrm{T}} > 0 \tag{8}$$

*Passivity of the parameter estimation algorithm in the B3 block*

$$\langle -v, Y\tilde{\theta}\rangle_T = \int_0^T -v^{\mathrm{T}}Y\tilde{\theta}\,dt$$

$$= \int_0^T \dot{\tilde{\theta}}\Gamma^{-1}\tilde{\theta}\,dt \quad \geqslant -\frac{1}{2}\tilde{\theta}^{\mathrm{T}}(0)\Gamma^{-1}\tilde{\theta}(0) \tag{9}$$

Where we simply use from (8) that:

$$v^{\mathrm{T}}Y = -\dot{\tilde{\theta}}^{\mathrm{T}}\Gamma^{-1}$$

So the mapping $-v \to Y\tilde{\theta}$ is passive.



*Strict passivity of the B1 block.* It is sufficient that $H$ be strictly passive for these adap laws to ensure the global asymptotic convergence of the tracking error. This is achieved choosing in each case:

*(1) Scheme proposed in Reference 7.* In this case

$$H(\tilde{q}, v) = F_v v + F_p\tilde{q}, \qquad \text{with } F_p, F_v > 0$$

so there exists a $\delta > 0$ such that

$$\langle v, F_v v + F_p\tilde{q}\rangle_T = \int_0^T (v^{\mathrm{T}}F_v v + v^{\mathrm{T}}F_p\tilde{q})\,dt \geqslant \delta\|v\|_{2,T}^2 - \frac{1}{2}\tilde{q}^{\mathrm{T}}(0)F_p\tilde{q}(0)$$

where $\|\ \|_{2,T}$ denotes the truncated $L_2$ norm.

*(2) Scheme proposed in Reference 8.* This time

$$H(\tilde{q}, v) = H(v) = \mu H^*(x)v$$

where $\mu > 0$, $H^*(x) = J^{-\mathrm{T}}H(q)J^{-1}$ and $x$ represents the cartesian co-ordinates. $J$ is the sq Jacobian matrix of the manipulator, assumed to be full-rank. In this case one needs following additional assumption:

$$\text{there exists } \alpha > 0 \text{ such that } \alpha I \leqslant H^*(x)$$

Then

$$\langle v, \lambda H^*(x).v\rangle_T = \int_0^T v^{\mathrm{T}}\lambda H^*(x)v\,dt \geqslant \lambda\alpha\|v\|_{2,T}^2$$

*Remark.* In case (2), the authors considered a unified approach of motion and force con in the cartesian space. The signal $v$ is then defined by the following relation:

$$\dot{v} + \lambda v = -R[G(s)S\xi + (I-S)F(s)R^{-1}\bar{F}]$$

where $\lambda$ is a positive scalar, $\xi$ is related to the tracking error $e = x_d - x$ and the measured f $F$ by

$$\xi = R^{-1}e + \Lambda G(s)^{-1}R^{-1}F$$

Where $\Lambda$ is a selection matrix: $\Lambda = 1$ or 0, $R$ is a constant matrix, $G(s)$ and $F(s)$ are tran functions of degree 2, and $S$ is the selection matrix, which is diagonal with elements bei or 0: the axis motion-controlled correspond to the '1', the others are force-controlled.

Then if $v \in L_2^n$, one can show that each component of $v = (v_1, v_2, \ldots, v_n)$ is $L_2$ as well, so lemma (Reference 16; p. 59) holds for each axis.

We are able to conclude

$$S\xi \text{ and } (I-S)R^{-1}\bar{F} \in L_2^n \cap L_\infty^n$$
$$S\dot{\xi} \text{ and } (I-S)R^{-1}\dot{\bar{F}} \in L_2^n$$

Therefore

$$S\xi \text{ and } (I-S)R^{-1}\bar{F} \text{ are continuous and tend towards zero as } t \to \infty$$

*(3) Scheme proposed in Reference 6.* In this case

$$H(v,\tilde{q}) = K_D v;\ K_D > 0$$

so there exists $\delta > 0$ such that:

$$\langle v, K_{\mathrm{D}}v\rangle_T = \int_0^T v^{\mathrm{T}}K_{\mathrm{D}}v \, \mathrm{d}t \geqslant \delta \| v \|_{2,T}^2$$

*System equivalent representation of schemes proposed in References 9 and 10*

In References 9 and 10 the authors have proposed a 'composite' adaptive law, with an update law utilizing both tracking and prediction errors. The control law is given as in (6) for $H(v, \tilde{q}) = K_{\mathrm{D}}v$. Replacing it in the robot dynamics (1), one obtains (7). The update law is given by:

$$\dot{\hat{\Theta}}(t) = -P(t)[Y^{\mathrm{T}}v + \alpha(t)W^{\mathrm{T}}e] \tag{10}$$

where $e$ is the prediction error: $e = \hat{\tau}_{\mathrm{f}} - \tau_{\mathrm{f}}$ with $\tau_{\mathrm{f}}$ the filtered torque obtained by

$$\tau_{\mathrm{f}} = W(q, \dot{q})\,\Theta$$

where $W(q, \dot{q}) = w(t)^{*}Y(q, \dot{q}, \ddot{q})$ and $w(t)$ is the impulse response of an exponentially stable, strictly proper filter.

Block B1 is identical to the previous case where $H(v, \tilde{q}) = K_{\mathrm{D}}v$, and therefore is strictly passive. So we have only to prove that the update law defines a passive subsystem: choosing the most general algorithm (called, in Reference 9, inherently-bounded-gain method) we get:

$$\frac{\mathrm{d}}{\mathrm{d}t}(P^{-1}(t)) = -\lambda(t)[P^{-1}(t) - K_0^{-1}] + \alpha(t)W^{\mathrm{T}}W \tag{11}$$

with $\lambda(t)$ and $\alpha(t)$ positive time-varying scalars; $K_0 > 0$.

$$\langle -v, Y\tilde{\theta}\rangle_T = \int_0^T -\tilde{\theta}^{\mathrm{T}}Y^{\mathrm{T}}v \, \mathrm{d}t = \int_0^T -\tilde{\theta}^{\mathrm{T}}[-P^{-1}\dot{\tilde{\theta}} - \alpha W^{\mathrm{T}}e] \, \mathrm{d}t$$

$$= \int_0^T \left\{\tilde{\theta}^{\mathrm{T}}\left[P^{-1}\dot{\tilde{\theta}} + \frac{\mathrm{d}}{\mathrm{d}t}(P^{-1})\tilde{\theta}\right] + \lambda\tilde{\theta}^{\mathrm{T}}(P^{-1} - K_0^{-1})\tilde{\theta}\right\} \mathrm{d}t$$

But

$$P^{-1}\dot{\tilde{\theta}} + \frac{\mathrm{d}}{\mathrm{d}t}(P^{-1})\tilde{\theta} = \frac{\mathrm{d}}{\mathrm{d}t}(P^{-1}\tilde{\theta})$$

It follows that the first term in the integral can be written as:

$$\tilde{\theta}^{\mathrm{T}}P^{-1}P\frac{\mathrm{d}}{\mathrm{d}t}(P^{-1}\tilde{\theta}) = u^{\mathrm{T}}P\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(u^{\mathrm{T}}Pu) - \frac{1}{2}u^{\mathrm{T}}\frac{\mathrm{d}P}{\mathrm{d}t}u$$

with $u = P^{-1}\tilde{\theta}$

Now using (11) and the following relation:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -P\frac{\mathrm{d}P^{-1}}{\mathrm{d}t}P$$

it follows that

$$u^{\mathrm{T}}\frac{\mathrm{d}P}{\mathrm{d}t}u = \lambda u^{\mathrm{T}}P[P^{-1} - K_0^{-1}]Pu - \alpha u^{\mathrm{T}}PW^{\mathrm{T}}WPu$$



We finally get

$$\langle -v, Y\tilde{\theta}\rangle_T \geqslant \int_0^T \left\{\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(u^{\mathrm{T}}Pu) + \frac{\lambda}{2}\tilde{\theta}^{\mathrm{T}}[P^{-1} - K_0^{-1}]\tilde{\theta}\right\} \mathrm{d}t$$

as $P^{-1} - K_0^{-1} \geqslant 0$ we obtain

$$\langle -v, Y\tilde{\theta}\rangle_T \geqslant -\frac{1}{2}\tilde{\theta}^{\mathrm{T}}(0)P^{-1}(0)\tilde{\theta}(0) \quad ($$

*Remark.* In Reference 9, the authors have considered separately several algorithms, from simple gradient method to the inherently-bounded-gain method. It is straightforward to that the IBG method is in fact the most general one, and that the other methods presented particular cases of the IBG. For instance, by choosing $\lambda(t) = 0$ and $\alpha(T) = 1$, one obtains standard-least-squares method; $K_0^{-1} = 0$ and $\lambda(t) = \lambda_0(1 - 1/k_0 \| P \|)$ leads to the g adjusted-forgetting method (GAF).

*System equivalent representation of the scheme proposed in Reference 11*

In Reference 11, the authors have proposed a so-called 'desired compensation adaptive l (DCAL), which is based on the fact that the regressor is no longer a function of measu signals, but of desired trajectories, i.e.

$$\dot{\hat{\Theta}} = -\Gamma Y^{\mathrm{T}}(q_{\mathrm{d}}, \dot{q}_{\mathrm{d}}, \ddot{q}_{\mathrm{d}})\,v = -\Gamma Y_{\mathrm{d}}v \quad ($$

(For simplicity we will denote $Y(q_{\mathrm{d}}, \dot{q}_{\mathrm{d}}, \ddot{q}_{\mathrm{d}})$ by $Y_{\mathrm{d}}$). It presents the advantage of being m computationally efficient, but leads to a more complex analysis.

The control law is given by

$$\tau = Y_{\mathrm{d}}\,\hat{\Theta} - F_{\mathrm{v}}v - F_{\mathrm{p}}\tilde{q} - f(v, \tilde{q}) \quad ($$

with $f(v, \tilde{q}) = \sigma_{\mathrm{n}} \| \tilde{q} \|^2 v$, $\quad \sigma_{\mathrm{n}}, F_{\mathrm{p}}$ and $F_{\mathrm{v}} > 0$

where $f(v, \tilde{q})$ is an auxiliary nonlinear feedback term introduced to compensate the kinem error due to $Y_{\mathrm{d}}$ in the update law. Replacing it in the robot dynamics leads to (7) with

$$H(v, \tilde{q}) = -F_{\mathrm{p}}\tilde{q} - F_{\mathrm{v}}v - f(v, \tilde{q}) - \Delta Y(v, \tilde{q}) \quad ($$

where $\Delta Y(v, \tilde{q}) = [Y(q, \dot{q}, \dot{a}, a) - Y_{\mathrm{d}}]\Theta$.

The update law is a gradient, and therefore is passive.

*Strict passivity of the block B1.* One can show that, choosing $F_{\mathrm{v}}, F_{\mathrm{p}}, \sigma_{\mathrm{n}}$, and $\lambda$ la enough, one can obtain a strictly passive operator (see Appendix A for details).

*System equivalent representation of schemes proposed in References 12 and 13*

First, let us introduce a slight modification in the passivity theorem presented in Sectio Assume that one is able to demonstrate passivity (or strict passivity), of any of the three blc B1, B2, B3 only for a given set of instants $S = (T_i)_{i \in N}$, and not for all $T > 0$. Then the res of Section 3 will still hold. Indeed, it is straightforward to conclude that

$$\| v \|_{2T_i} < \infty \qquad \text{for all } T_i \in S$$

Given any $T \geqslant 0$, there always exists a couple $(T_i, T_{i+1})$ such that $T_i \leqslant T \leqslant T_{i+1}$. Then

$$\int_0^{T_i} \| v(t) \|^2 \, dt \leqslant \int_0^{T} \| v(t) \|^2 \, dt \leqslant \int_0^{T_{i+1}} \| v(t) \|^2 \, dt < \infty$$

Then

$$\| v(t) \|_T < \infty \qquad \text{for all } T > 0 \Rightarrow v \in L_2^n$$

and the rest of the analysis follows.

In Reference 12, the authors considered a 'repetitive control law', designed for robots which are required to execute the same motion with a fixed period $t'$, i.e.

$$q_d(t + t') = q_d(t)$$
$$\dot{q}_d(t + t') = \dot{q}_d(t)$$
$$\ddot{q}_d(t + t') = \ddot{q}_q(t)$$

The robot is expected to 'learn' the trajectory after consecutive iterations from a period to another. The repetitive control law is given by

$$\tau = \hat{w}_r(t) - F_v v - F_p \tilde{q} - f(\tilde{q}, v) \tag{16}$$

Where the last terms are identical to those of Reference 11 in the 'DCAL' algorithm, and

$$w_r(t) = Y(q_d, \dot{q}_d, \ddot{q}_d)\theta = Y_d \theta \tag{17}$$

Replacing it into the robot dynamics leads to

$$M(q)\dot{v} + C(q, \dot{q})v = -F_v v - F_p \tilde{q} - \Delta Y(\tilde{q}, v) - f(\tilde{q}, v) + \tilde{w}_r(t) \tag{18}$$

where the blocks B1, B2, B3 clearly appear. Note that the B1 block is identical to the one in Reference 11 and can be made strictly passive by appropriate choice of the gains in (16).

*Passivity of block B3*. In Reference 12, the authors introduced a projection operation to keep the estimates between lower and upper bounds, which are assumed to be known *a priori*, thus ensuring boundedness of the estimated $\hat{w}_r(t)$.

During phases of adaptation, the estimation algorithm is given by

$$\tilde{w}_r(t) = \tilde{w}_r(t - t') - K_L v(t) \qquad K_L > 0$$

Assuming that there is no finite escape time and not taking into account the projection operation, one can show the passivity of the B3 block in this case too (see Appendix B for details).

*Remark*. The authors have also introduced[13] a modification in their adaptation algorithm:

$$\tilde{w}_r(t) = \tilde{w}_r(t - t') - K_L v(t - t')$$

to allow sufficient time for computation. It can be shown that, adding a signal $-F_{v2}v$ to the B3 block (separating the gain $F_v$ into two parts as done in Reference 13, this algorithm is also passive.

*System equivalent representation of schemes proposed in References 14a and 14b*

A new adaptation learning rule has been studied in Reference 14a. It presents the advantage that no projection operation is needed to keep the internal signals bounded. It is mainly based



on the fact that the unknown term $\tilde{w}_r(t)$ can be written as

$$\tilde{w}_r(t) = \int_0^{t'} K(t, \tau)\tilde{c}(t, \tau) \, d\tau$$

where $c(t, \tau)$ is an influence function, and $K(t, \tau)$ is a positive non-degenerate kernel Reference 14 for details). The update law is given by

$$\frac{\partial}{\partial t} \hat{c}(t, \tau) = -K_L K(t, \tau) v(t), \qquad K_L = K_L^T > 0$$

It is then possible to show that this learning rule is also passive (see Appendix C).

In order to reduce the on-line computations, the authors have also introduced a so-ca delayed repetitive learning rule.[14b] The influence function is now given by

$$\hat{c}_{k+1} = \hat{c}_k + \int_{(k-1)t'}^{kt'} K_L K(\sigma, \tau) v(\sigma) \, d\sigma$$

and the update law:

$$\hat{w}_r(t) = \int_0^{t'} K(t, \tau)\hat{c}_k(\tau) \, d\tau$$

for $kt' \leqslant t < (k+1)t'$, $k = 1, 2, 3, \ldots$.

Assuming this time again there is no finite escape time (at least during the first pe $0 \leqslant t < t'$), then one can show that this delayed algorithm is passive (see Appendix D).

## 5. STABILITY ANALYSIS

The foregoing studies have shown the global asymptotic convergence of the tracking erro the schemes considered. In order to complete the stability result, it only remains to state all the signals in the equivalent feedback representation, including the estimates, are bounc Similarly to what has been done in Reference 17, let us consider the following function:

$$V(T) = \gamma + \langle -v, \tilde{\tau} \rangle_T + \beta + \langle -v, Y\tilde{\theta} \rangle_T + \mu + \langle v, H(v, \tilde{q}) \rangle_T$$

Note that choosing $\gamma, \beta$ and $\mu$ adequately ensures that $V$ is a positive function, as we I shown previously that there exist $\gamma > 0, \beta > 0, \mu > 0$ and $\delta > 0$ such that for all $T > 0$

$$\langle -v, \tilde{\tau} \rangle_T \geqslant -\gamma, \langle -v, Y\tilde{\theta} \rangle_T \geqslant -\beta \qquad \text{and} \qquad \langle v, H(v, \tilde{q}) \rangle_T \geqslant \delta \| v \|_{2,T}^2 - \mu \geqslant -\mu$$

Choosing $\gamma = 1/2 v^T(0) M(q(0)) v(0)$, $V(T)$ can be rewritten as

$$V(T) = \frac{1}{2} v^T M(q) v + \beta + \int_0^T (-v^T Y\tilde{\theta}) \, dt + \mu + \int_0^T v^T H(v, \tilde{q}) \, dt$$

Differentiating $V$ along the trajectories of the system leads to

$$\dot{V}(T) = 0 \qquad \text{for all } T \geqslant 0$$

Hence one gets $V(T) = V(0)$ so $V \in L_\infty$ and then $V \in L_\infty^n$. Applying a similar result to the len in Section 3 (see Reference 17, p. 59) we get

$$\dot{\tilde{q}} \in L_\infty^n$$

and then $q, \dot{q}, a = a(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d)$, $a = \dot{q} - v$ and $Y(q, \dot{q}, a, \dot{a})$ remain $L_\infty$-bounded.

*(1) Schemes proposed in References 6–8 and 11.* In these cases

$$\beta + \langle -v, Y\tilde{\theta}\rangle_T \geqslant \tfrac{1}{2}\tilde{\theta}^{\mathrm{T}}(T)\Gamma\tilde{\theta}(T) \qquad \text{so } \tilde{\theta} \in L_\infty^r \text{ and } Y\tilde{\theta} \in L_\infty^n$$

(where we have chosen $\beta = \frac{1}{2}\tilde{\theta}^{\mathrm{T}}(0)\Gamma\tilde{\theta}(0)$).

*(2) Schemes proposed in References 9 and 10.* In this case one gets

$$\beta + \langle -v, Y\tilde{\theta}\rangle_T = \int_0^T \left\{\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(u^{\mathrm{T}}Pu) + \frac{\lambda}{2}\tilde{\theta}^{\mathrm{T}}[P^{-1} - K_0^{-1}]\tilde{\theta}\right\}\mathrm{d}t$$

$$\geqslant \frac{1}{2}\tilde{\theta}^{\mathrm{T}}(T)P^{-1}(T)\tilde{\theta}(T) \geqslant 0 \qquad (\text{as } P^{-1} - K_0^{-1} \geqslant 0)$$

(where we have chosen $\beta = \frac{1}{2}\tilde{\theta}^{\mathrm{T}}(0)P^{-1}(0)\tilde{\theta}(0)$). Hence

$$\tilde{\theta} \in L_\infty^r \text{ and } Y\tilde{\theta} \in L_\infty^n$$

*Remark.* We have shown that the closed-loop error equation can in each case be written as follows:

$$M(q)\dot{v} + C(q,\dot{q})v + H(v,\tilde{q}) = Y\tilde{\theta}$$

Then

$$\dot{v} = [M(q)]^{-1}[Y\tilde{\theta} - H(v,\tilde{q}) - C(q,\dot{q})v]$$

As the right-hand side of the expression above is bounded, one gets

$$\dot{v} \in L_\infty^n$$

Now, $\dot{\tilde{q}} = H(s)\dot{v}$; applying lemma (Reference 17, p. 59) one concludes:

$$\dot{\tilde{q}} \in L_\infty^n \cap L_2^n$$
$$\ddot{\tilde{q}} \in L_\infty^n$$
$$\dot{\tilde{q}} \text{ is continuous and } \dot{\tilde{q}} \to 0 \text{ as } t \to +\infty$$

*(3) Scheme proposed in Reference 12.* Utilizing the same technique, we here replace $Y\tilde{\theta}$ by $\tilde{w}_\mathrm{r}(t)$. Taking

$$\beta = \int_0^{t'} \|\tilde{w}_\mathrm{r}(t)\|_{K_\mathrm{I}^{-1}}^2\,\mathrm{d}t + \alpha$$

we obtain

$$\beta + \langle -v, \tilde{w}_\mathrm{r}\rangle_{T_n} \geqslant \int_{(n-1)t'}^{nt'} \|\tilde{w}_\mathrm{r}(t)\|_{K_\mathrm{I}^{-1}}^2\,\mathrm{d}t \geqslant 0 \qquad \text{for all } n \geqslant 2, \text{ with } T_n = nt'$$

It shows that $\tilde{w}_\mathrm{r}$ is $L_2[(n-1)t', nt']$ bounded. Hence, under the assumption that the algorithm does not diverge during the first period $0 \leqslant t \leqslant t'$, there is no finite escape time in the system. Unfortunately, $L_\infty$-boundedness cannot be ensured without a projection operation as done in Reference 12.

Moreover, boundedness of the several signals previously stated can no longer be ensured for all $t \geqslant 0$, as $V(T)$ is a positive definite function only for $T = T_n, n \in N^*$. Note that as lemma in Section 3 still holds, one still gets

$$\tilde{q} \in L_\infty^n$$

Nevertheless, boundedness of the other signals for all $t \geqslant 0$ can be shown through a Lyapunov-type approach, as done in Reference 12.



*(4) Scheme proposed in Reference 14a.* We now choose

$$\beta = \frac{1}{2}\int_0^{t'} \tilde{c}(0,\tau)^{\mathrm{T}}K_\mathrm{I}^{-1}\tilde{c}(0,\tau)\,\mathrm{d}\tau$$

Hence

$$\int_0^{t'} \|\tilde{c}(T,\tau)\|^2\,\mathrm{d}\tau \in L_\infty \qquad \text{for all } T > 0$$

Following what has been done in Reference 14, this allows us to conclude on the boundec of the estimated signals $w_\mathrm{r}(t)$. Note also that, contrary to Reference 12, the analysis is for all $T > 0$, and is not restricted to a set of discrete instants $T_n$. So the foregoing conclu on the schemes of References 6–11 are available.

*(5) Scheme proposed in Reference 14b.* As for Reference 12, the passivity analysis o update law has been done for a set of discrete instants $T_n$, and not for all $T > 0$. S conclusions are equivalent. However, because of the particular form of the update law (21 (22), and using the fact that $v \in L_2$, the estimates are bounded.

*Remark.* Concerning adaptation algorithms, one can note that the gradient method is spread in robot adaptive control, as can easily be seen as a passive map between its input output. In Reference 15, the authors have exhibited a class of passive adaptation algorit known as the 'integral + proportional PAA', which had been first proposed in Referenc but never applied in the field of robot control.

In fact, though least-squares algorithms are recognized as ensuring better performances the gradient method, one is only able to establish the passivity of the latter. In Referenc the authors have recently introduced a modified least-squares estimation algorithm base passivity properties: Their control law is given by

$$\tau = \hat{M}(q)\dot{a} + \hat{C}(q,\dot{q})a + \hat{g}(q) - Kv$$

$a$ resulting from an outloop control law as in Section 4 of this paper (see References 24 19 for further details). Replacing it in the robot dynamics leads to

$$M(q)\dot{v} + C(q,\dot{q})v = Y(q,\dot{q},a,\dot{a})\tilde{\theta} - Kv$$

The modified least-squares algorithm is given by

$$\hat{\theta} = \hat{\theta}_\mathrm{L} + s \text{ and } \dot{\hat{\theta}}_\mathrm{L} = -PY^{\mathrm{T}}v$$

with

$$s = \frac{-Y^{\mathrm{T}}}{1 + \mathrm{tr}(Y^{\mathrm{T}}Y)}\frac{v}{1 + v^{\mathrm{T}}v}(\hat{\theta}_\mathrm{L}^{\mathrm{T}}A\hat{\theta}_\mathrm{L} + M(1 + \lambda\lambda_{\max}(R)))$$

$$\dot{P} = \alpha(t)\left[-P\left\{\frac{Y^{\mathrm{T}}Y}{1 + \mathrm{tr}(Y^{\mathrm{T}}Y)} + \lambda R\right\}P + \lambda P\right]$$

$$\alpha(t) = \frac{v^{\mathrm{T}}YY^{\mathrm{T}}v}{(1 + v^{\mathrm{T}}v)[1 + \mathrm{tr}(Y^{\mathrm{T}}Y)]}$$

$$A = \frac{Y^{\mathrm{T}}Y}{1 + \mathrm{tr}(Y^{\mathrm{T}}Y)} + \lambda R$$

with $\lambda \geqslant 0$ and $R > 0$; $\lambda_{\min}(R) \leqslant P^{-1}(0) \leqslant \lambda_{\max}(R) + 1/\lambda$, where $s$ is the modification to the least squares estimate that renders the mapping $-v \to Y\tilde{\theta}$ passive (see References 19 and 24 for details).

## 6. CONCLUSION

In this paper, we have shown that most of the direct continuous adaptive control laws proposes to date for rigid manipulators can be analysed through a passivity approach, and that the simple framework proposed enables simplifications to be made of certain analyses and provides a clear understanding of such algorithms.

One can now legitimately expect this different point of view to result in new algorithms designed directly from the passivity theorem. In fact, it has been shown that the passivity approach can lead to new adaptive control laws in the case of hybrid force/position control (see, for example, References 22 and 23 where extensions of the motion control laws in References 6 and 8 respectively have been proposed). Note also that some adaptive schemes recently proposed in the literature (see, for example, Reference 25) can be analysed through a passive point of view, although their structure slightly differs from the one presented here (namely, no use of property (3) in Section 2 is done in Reference 25). It has also to be emphasized that the study in this paper can be viewed as a particular (but important) case of the more general study concerning relationships between Lyapunov functions and passivity tools presented in Reference 26.

## APPENDIX A: *Strict passivity of the B1 block*

In this case, we can write

$$\langle -v, H(v,\tilde{q})\rangle_T = \int_0^T v^{\mathrm{T}}[F_v v + F_p\tilde{q} + \sigma_n \|\tilde{q}\|^2 v + \Delta Y(v,\tilde{q})]\,dt$$

The proof mainly hinges on the following relation (see Reference 11 for details):

$$v^{\mathrm{T}}\Delta Y(v,\tilde{q}) \geqslant -v^{\mathrm{T}}[\lambda M(q) + b_1 I]v - v^{\mathrm{T}}[-\lambda^2 M(q) + b_2 I]\tilde{q} - b_3[\|v\|^2\|\tilde{q}\| + \lambda\|v\|\,\|\tilde{q}\|^2]$$

with

$$b_1 = b_1(\dot{q}_d) \geqslant 0$$
$$b_2 = b_2(\dot{q}_d, \ddot{q}_d) \geqslant 0$$
$$b_3 \geqslant 0$$

Then we can write:

$$\langle v, H(v,\tilde{q})\rangle_T \geqslant \int_0^T \{\sigma_v v^{\mathrm{T}} v + \sigma_p v^{\mathrm{T}}\tilde{q} + \sigma_n\|\tilde{q}\|^2\|v\|^2 - v^{\mathrm{T}}[\lambda M(q) + b_1 I]v\}\,dt$$
$$- \int_0^T \{v^{\mathrm{T}}[\lambda^2 M(q) + b_2 I]\tilde{q} - b_3[\|v\|^2\|\tilde{q}\| + \|v\|\,\|\tilde{q}\|^2]\}\,dt$$

As it has been done in Reference 11, the last term in the integral can be written:

$$b_3[\|v\|^2\|\tilde{q}\| + \lambda\|v\|\,\|\tilde{q}\|^2] = \frac{b_3}{4}\|v\|^2 + \lambda\frac{b_3}{4}\|\tilde{q}\|^2 - b_3\|v\|^2\left[\frac{1}{2} - \|\tilde{q}\|\right]^2$$
$$- \lambda b_3\|\tilde{q}\|^2\left[\frac{1}{2} - \|v\|\right]^2 + (1+\lambda)\,b_3\|v\|^2\|\tilde{q}\|^2$$



Grouping similar quadratic terms together,

$$\langle v, H(v,\tilde{q})\rangle_T \geqslant \int_0^T v^{\mathrm{T}}\left[\left(\sigma_p - b_1 - \frac{b_3}{4}\right)I - \lambda M(q)\right]v\,dt + \int_0^T \|v\|^2\|\tilde{q}\|^2[\sigma_n - (1+\lambda)\,b_3]\,dt$$
$$+ \int_0^T \left\{v^{\mathrm{T}}[(\sigma_p - b_2)I + \lambda^2 M(q)]\tilde{q} - \lambda\frac{b_3}{4}\|\tilde{q}\|\right.$$

But

$$v^{\mathrm{T}}[(\sigma_p - b_2)I + \lambda^2 M(q)]\tilde{q} = \lambda_p^2\sigma_p\|\tilde{q}\|^2 + \sigma_p\tilde{q}^{\mathrm{T}}\dot{\tilde{q}} + v^{\mathrm{T}}[\lambda^2 M(q) - b_2 I]\tilde{q}$$
$$\geqslant \lambda\sigma_p\|\tilde{q}\|^2 + \frac{\sigma_p}{2}\frac{d}{dt}(\tilde{q}^{\mathrm{T}}\tilde{q}) - \frac{1}{2}v^{\mathrm{T}}[\lambda^2 M(q) - b_2 I]v - \frac{1}{2}\tilde{q}^{\mathrm{T}}[\lambda^2 M(q) - l$$

Finally we obtain

$$\langle v, H(v,\tilde{q})\rangle_T \geqslant \int_0^T v^{\mathrm{T}}\left[\left(\sigma_v - b_1 + \frac{b_2}{2} - \frac{b_3}{4}\right)I - \lambda M(q) - \frac{\lambda^2}{2}M(q)\right]v\,dt$$
$$+ \int_0^T \tilde{q}^{\mathrm{T}}\left[\left(\lambda\sigma_p + \frac{b_2}{2} - \lambda\frac{b_3}{4}\right)\left(I - \frac{\lambda^2}{2}M(q)\right)\right]\tilde{q}\,dt$$
$$+ \int_0^T \|v\|^2\|\tilde{q}\|^2[\sigma_n - (1+\lambda)b_3]\,dt - \frac{\sigma_p}{2}\|\tilde{q}(0)\|^2$$

We are now able to claim that, provided $F_p$, $F_v$, $\sigma_n$ and $\lambda$ are chosen so that the following relatior satisfied:

$$F_p = \sigma_p I \geqslant \frac{1}{\lambda}\left[\left(-\frac{b_2}{2} + \lambda\frac{b_3}{4}\right)I + \frac{\lambda^2}{2}M(q)\right]$$
$$F_v = \sigma_v I \geqslant \left(b_1 - \frac{b_2}{2} + \frac{b_3}{4}\right)I + \lambda\left(1 + \frac{\lambda}{2}\right)M(q)$$
$$\sigma_n \geqslant (1+\lambda)b_3$$

Then the block B1 is a passive one. Moreover, if $\sigma_v$ is chosen large enough, there exists a $\delta > 0$ such

$$\langle v, H(v,\tilde{q})\rangle_T \geqslant \delta\|v\|_{2,T}^2 - \mu \qquad \text{with } \mu = \frac{\sigma_p}{2}\|\tilde{q}(0)\|^2$$

and the block B1 is then strictly passive.

## APPENDIX B: *Passivity of the B3 block*

For all $T_n = nt'$, $n \in N^*$, one gets

$$\langle -v(t), \tilde{w}_r(t)\rangle_{T_n} = \sum_{k=0}^{n-1}\int_{kt'}^{(k+1)t'} -v^{\mathrm{T}}\tilde{w}_r(t)\,dt = \sum_{k=0}^{n-1}\int_{kt'}^{(k+1)t'} -v^{\mathrm{T}}\tilde{w}_r\,dt - \int_0^{t'} v^{\mathrm{T}}\tilde{w}_r\,dt$$

We will assume now that there is no finite escape time in this algorithm so that the last integral te the expression above is bounded:

$$\text{there exists } \alpha > 0 \text{ such that } \left|\int_0^{t'} v^{\mathrm{T}}\tilde{w}_r\,dt\right| \leqslant \alpha < \infty$$

Then

$$\langle -v(t), \tilde{w}_r(t)\rangle_{T_n} \geqslant \sum_{k=1}^{n-1}\int_{kt'}^{(k+1)t'} -v^{\mathrm{T}}\tilde{w}_r(t)\,dt - \alpha$$

As it has been done in Reference 12, the term under the integral can be rewritten

$$-v^{\mathrm{T}}(t)\tilde{w}_r(t) = \frac{1}{2}[\tilde{w}_r^{\mathrm{T}}(t)K_L^{-1}\tilde{w}_r^{\mathrm{T}}(t) - \tilde{w}_r^{\mathrm{T}}(t-t')K_L^{-1}\tilde{w}_r(t-t') + v^{\mathrm{T}}(t)K_L v(t)]$$

Replacing this expression in the integral leads to

$$\langle -v, \tilde{w}_r\rangle_{T_n} \geqslant \sum_{k=1}^{n-1} \int_{kt'}^{(k+1)t'} \{\| \tilde{w}_r(t) \|^2_{K_L^{-1}} - \| \tilde{w}_r(t-t') \|^2_{K_L^{-1}}\} \, dt - \alpha$$

Introducing the following variable change in the second term in the integral

$$u = t - t' \qquad du = dt$$

one gets

$$\langle -v, \tilde{w}_r\rangle_{T_n} \geqslant \sum_{k=1}^{n-1} \left[ \int_{kt'}^{(k+1)t'} \| \tilde{w}_r(t) \|^2_{K_L^{-1}} \, dt - \int_{(k-1)t'}^{kt'} \| \tilde{w}_r(u) \|^2_{K_L^{-1}} \, du \right] - \alpha$$

Noticing that all the terms under the integrals cancel but for the following ones:

$$\int_{(n-1)t'}^{nt} \| \tilde{w}_r(t) \|^2_{K_L^{-1}} \, dt \quad \text{and} \quad \int_0^{t'} \| \tilde{w}_r(t) \|^2_{K_L^{-1}} \, dt$$

we obtain

$$\langle -v, \tilde{w}_r\rangle_{T_n} \geqslant \int_{(n-1)t'}^{nt'} \| \tilde{w}_r(t) \|^2_{K_L^{-1}} \, dt - \int_0^{t'} \| \tilde{w}_r(u) \|^2_{K_L^{-1}} \, du - \alpha$$

$$\geqslant - \int_0^{t'} \| \tilde{w}_r(u) \|^2_{K_L^{-1}} \, du - \alpha$$

Making the same assumption on the first period as previously, one sees that the estimation algorithm is passive for all $T_n$.

## APPENDIX C: *Passivity of the B3 block*

$$\langle -v, \tilde{w}_r\rangle_T = \int_0^T - v^{\mathrm{T}} \int_0^{t'} K(t,\tau)\tilde{c}(t,\tau) \, d\tau \, dt = \int_0^T \int_0^{t'} \left[\frac{\partial}{\partial t}\tilde{c}(t,\tau)\right]^{\mathrm{T}} K_L^{-1}\tilde{c}(t,\tau) \, d\tau \, dt$$

which follows from the fact that

$$\frac{\partial}{\partial t}\tilde{c}(t,\tau) = \frac{\partial}{\partial t}\hat{c}(t,\tau)$$

Then we obtain:

$$\langle -v, \tilde{w}_r\rangle_T = \frac{1}{2}\left\{\int_0^{t'} \tilde{c}(T,\tau)^{\mathrm{T}} K_L^{-1}\tilde{c}(T,\tau) \, dt - \int_0^{t'} \tilde{c}(0,\tau)^{\mathrm{T}} K_L^{-1}\tilde{c}(0,\tau) \, d\tau\right\}$$

$$\geqslant -\frac{1}{2}\int_0^{t'} \tilde{c}(0,\tau)^{\mathrm{T}} K_L^{-1}\tilde{c}(0,\tau) \, d\tau$$

$$\geqslant -\frac{1}{2\sigma_{\max}(K_L)}\int_0^{t'} \| \tilde{c}(0,\tau) \|^2 \, d\tau$$

Hence, with the assumption that no finite escape occurs during the first time period, the update law defines a passive subsystem.

## APPENDIX D:

$$\langle -v, \tilde{w}_r\rangle_{T_n} = \int_0^{t'} -v^{\mathrm{T}}\tilde{w}_r \, dt - \int_{t'}^{T_n} -v^{\mathrm{T}}\tilde{w}_r \, dt$$

with $T_n = nt'$.

The first term of the left-hand side is bounded. We shall now focus on the second one.

$$-\int_{t'}^{T_n} -v^{\mathrm{T}}\tilde{w}_r \, dt = -\sum_{k=2}^{n} \int_0^{t'} \tilde{c}_{k-1}^{\mathrm{T}}(\tau) \int_{(k-1)t'}^{kt'} K(t,\tau)v(t) \, dt \, d\tau$$



Now

$$-\tilde{c}_{k-1}(\tau) = -\tilde{c}(1) + \sum_{i=1}^{k-2} \int_{(i-1)t'}^{it'} K_L K(\tau,\sigma)v(\sigma) \, d\sigma$$

Let us define

$$x(i) = \int_{(i-1)t'}^{it'} K_L^{1/2} K(\tau,\sigma)v(\sigma) \, d\sigma$$

Then we obtain:

$$\int_{t'}^{T_n} -v^{\mathrm{T}}\tilde{w}_r \, dt = \int_0^{t'} \sum_{k=2}^{n} x^{\mathrm{T}}(k)\left[\sum_{i=1}^{k-2} x(i) - K_L^{-1/2}\tilde{c}_1\right] d\tau$$

Now we use the fact that:

$$\sum_{k=2}^{n} x^{\mathrm{T}}(k)\left[\sum_{i=1}^{k-2} x(i) - K_L^{-1/2}\tilde{c}_1\right] = \sum_{k=1}^{n} x^{\mathrm{T}}(k)\left[\sum_{i=1}^{k} x(i) - K_L^{-1/2}\tilde{c}_1\right]$$

$$- \sum_{k=2}^{n} x^{\mathrm{T}}(k)[x(k) + x(k-1)] - x^{\mathrm{T}}(1)[x(1) - K_L^{-1}$$

and the following lemma.[20]

*Lemma*

Given a sequence of real vectors $x(k)$ and a constant vector $c$, then the following relation is tru

$$\sum_{k=1}^{n} x^{\mathrm{T}}(k)\left[\sum_{i=1}^{k} x(i) + c\right] \geqslant -\frac{1}{2}c^{\mathrm{T}}c$$

For all $n \geqslant 1$.

Hence we obtain

$$\langle -v, \tilde{w}_r\rangle_{T_n} \geqslant -a - \int_0^{t'} \left\{\tilde{c}_1^{\mathrm{T}} K_L^{-1}\tilde{c}_1 + \frac{3}{2}x^{\mathrm{T}}(1)x(1)\right\} d\tau - \int_0^{t'} \sum_{k=2}^{n} \left\{x^{\mathrm{T}}(k)x(k) + x^{\mathrm{T}}(k)x(k-1)\right\} d$$

The second term of the right-hand side is bounded. Recalling the definition of $x(k)$, the third term be written as:

$$-\int_0^{t'} \sum_{k=2}^{n} \{x^{\mathrm{T}}(k)x(k) + x^{\mathrm{T}}(k)x(k-1)\} \, d\tau \geqslant -2\sum_{k=1}^{n} \int_{(k-1)t'}^{kt'} v^{\mathrm{T}}(t)\kappa K_L v(t) \, dt$$

where

$$\kappa = \int_0^{t'} \| K(\tau,\sigma) \|^2 \, d\tau$$

(see References 13 and 14 for further details). Then we get

$$\langle -v, \tilde{w}_r\rangle_{T_n} \geqslant -\alpha - \gamma - 2\sum_{k=1}^{n} \int_{(k-1)t'}^{kt'} v^{\mathrm{T}}(t)\kappa K_L v(t) \, dt$$

where $\alpha$ and $\gamma$ stand for the upper bounds of the initial terms. One can easily see that it is sufficient to add an auxiliary signal $-F_{v2}v$ to $\tilde{w}_r$ in order to obtain a passive subsystem, provided the follo condition is fulfilled: $F_{v2} - 2\kappa K_L > 0$.

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# ROBUST CONTROL OF ROBOTIC MANIPULATORS WITHOUT VELOCITY FEEDBACK

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## SUMMARY

This study concerns the problem of robust control of robotic manipulators without joint ve feedback. A robust lead + bias controller is studied. The bias signal is intended to compensat nonlinear dynamics of the robot. The focus of this study is robustness when the nonlinear compen is not perfect and the external disturbances are not negligible.

A conservative polynomial bound is introduced to describe the worst feedback effect o compensation error and the external disturbances. The polynomial bound covers a class of possibl signals, synthesized according to the available knowledge about the robot dynamics. Based o polynomial bound, the tracking errors of a lead + bias controller are proved to be uniformly bou They can be minimized by a proper design of the bias signal. In the ideal case where the bias compensates the robot dynamics perfectly, the tracking errors will converge to zero.

## 1. INTRODUCTION

Robust tracking control of robotic manipulators has been studied by many researchers.[1] of the reported controllers require complete state feedback to provide stable tracking fc closed-loop system, which means that both position and velocity must be measured at joint. While the joint positions can be measured very accurately by encoders, the joint ve measurements are often contaminated by noise, due to the less accurate nature of tachom To overcome this problem, some researchers proposed nonlinear observers for joint ve estimation. A sliding observer for general nonlinear systems was studied by Slotine (1987);[2] the first observer for robotic systems was proposed by Canudas de Wit and Sl (1989);[3] Nicosia *et al.*[4–6] studied a number of nonlinear observers for nonlinear system elastic robots; observers plus controllers were studied by Nicosia and Tomei (1990) Canudas de Wit *et al.* (1990).[8] More recently, robust nonlinear smooth observers have reported by Canudas de Wit *et al.*[9,10] All these works have a common objective: robust cc of robotic manipulators without direct measurement of joint velocities.

In this paper, a different approach is investigated. Instead of trying to estimate the ve by observers, the high-pass filtered position feedback is used as a substitute for the ve feedback. In other words, a lead + bias controller is applied to robotic manipulators. Th compensator is synthesized by the traditional technique for linear time-invariant systems the bias signal is synthesized by feed-forward dynamics. When the robot parameters ar correct, the bias signal will be inaccurate. In order to cover a large class of admissibl

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