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Force Measurement Time-Delays and Contact Instability Phenomenon

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In this paper we consider stability issues of manipulators in contact with a rigid environment, i.e. submitted to a unilateral constraint of the form $\mathcal{F}(q) \geq 0$, *where* q *is the vector of generalised coordinates. The interaction force feedback loop contains time-delays which may induce instability phenomena. Sufficient delay-dependent conditions are derived to guarantee that the robot's tip remains in contact with the surface. These conditions are found by taking into account the fact that, due to the unilateral nature of the constraint, the interaction force must have constant sign during the whole task. We analyse the cases of proportional and proportional-integral force feedback.*

Keywords: Force control; Holonomic constraints; Lyapunov functional; Neutral functional differential equation; Stability; Time-delay

1. Introduction

Robot manipulator force control has motivated many research studies during the last 15 years. One of the first papers dealing with so-called hybrid control of manipulators subject to holonomic constraints was published in Raibert and Craig [30]. Since that time, many researchers in the field have extended, discussed and improved the pioneering work in Raibert and Craig [30]; see, for example, MacClamroch and Wang [23], Yoshikawa [39], Wen and Murphy [37], Yun [41], Sinha and Goldenberg [31], Duffy [13], Fisher and Shahid Mujtaba [16], De Luca and Manes [12] and Mills [25,26] to cite only a few.

Contact instability [10] and performance issues of force controllers have been investigated in detail in Visher and Khatib [33], An and Hollerbach [1], Volpe and Khosla [34], Eppinger and Seering [14], Wen and Murphy [37], Kazerooni [19], An [1], Mills [26], Anderson and Spong [3] and Niemeyer and Slotine [29]. The effects of *non-colocation* of sensors and actuators have been studied in Eppinger and Seering [14] and in Colgate and Hogan [10]. Kazerooni [19] and An [1] have shown that unmodelled dynamics may also yield instability. Mills [26] shows that small flexibilities in the joints do not destabilise the closed-loop system when a controller for rigid robots is designed. Wen and Murphy [37] argue that if an integral force feedback is used, then integral gain should be chosen very small due to possible flexibilities in the mechanical system. A thorough study of force control strategies can be found in Volpe and Khosla [35], where it is concluded that integral force feedback is the best basic strategy for force control of manipulators. Proportional force feedback is proved to be a suitable strategy for the transition phase in Volpe and Khosla [34].

Some work in teleoperation systems has focused on contact instability when time-delays are present in the force feedback loop: Anderson and Spong [3] used scattering theory and modelled time delays as a passive transmission line; Niemeyer and Slotine

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[29] used the idea of impedance matching to supress reflections from the boundaries between the transmission line and the master and slave manipulators. Few studies have been devoted to force measurement time-delays in the field of manipulator control (hybrid force/position control) [30]. Among these, Wen and Murphy [37] and Wilfinger et al. [38] provide a short study of delay effects in proportional force feedback, and argue that integral force feedback robustifies the closed-loop system. Fiala and Lumia [15] studied the consequences of time-delays in PD motion controllers when the robot was in contact with a compliant environment. It is noteworthy that the unilaterality of the constraints is not taken into account in these studies. Our main goal in this note is to examine the effects of time-delays in measured variables (in particular the interaction force between the robot's tip and environment supposed rigid) on stability of a force control scheme.

Basically, one can make two assumptions concerning the environment within which the robot interacts: either it is *rigid*, in which case the manipulator is to be considered as a mechanical system with holonomic constraints [39]; or it is *compliant*, i.e. the interaction force F and the environment's deformation q verify an equation like $F = q$ (if the environment acts like a spring with stiffness matrix K); here F and q are six-dimensional vectors (F is a wrench) expressed in the so-called task-frame, and thus contain force (translations) and torques (rotations) [8,22].

Force measurement delays have been experimentally evidenced in Wilfinger et al. [38] and Chiu and Lee [9]. Thus, for example, in the latter a transition phase controller sufficiently robust to force sensor delays and parametric uncertainties is presented.

In this paper, we shall consider the very simple example of a 1 degree of freedom (d.f.) prismatic manipulator (i.e. a mass m) that contacts a 1 d.f. environment. Notice that the study of such a simple model proves to be sufficient in higher-dimensional cases when the non-linear robot dynamics are decoupled and compensated for [27,39]; moreover, it should be noted that problems on force control of manipulators are often studied and understood with 1 d.f. models (masses, springs, dampers) ([14,34] and references therein). We shall treat only the *rigid* environment *case*.

The dynamical equations are given by:

$$\begin{cases} m\ddot{q}(t) = U & \text{if } q > 0 \\ F + U = 0 & \text{if } q \equiv 0 \end{cases} \tag{1}$$

where U is the (force) control input and F is the interaction force between the mass and the environment. The first equation in (1) characterises the *free motion*, whereas the second one characterises *constrained motion*.

Remark 1. Note that assuming $F = -kq$ and that velocity $\dot{q}$ is available for feedback is equivalent to allowing the designer to use the force derivative $\dot{F}(t) = -k\dot{q}(t)$ in the controller (provided k is known). This would, however, require differentiation of the measured force F_m in the rigid case and this is not a practically feasible procedure, since F_m is often corrupted by noise [34]. Thus, as we shall see later, closed-loop equations for the rigid case will typically contain F together with its successive time integrals [40].

Since we are interested in the closed-loop equations when the measured quantities contain time-delays, these equations will typically become functional differential equations of neutral type (NFDE), which can be characterised by the fact that the rate of change of state depends not only on the past states, but also on the past rates of state [18,20].

There has been special interest in the study of NFDE in the last 20 years [7,11,17,20,21,24,32] since these 'special' differential equations are frequently encountered in the modelling of unsteady motion of the elastic flying vehicle (as for example the aeroautoelasticity models) or in the lossless transmission lines ([18,21] and references therein).

The stability criteria can be classified into two classes: frequency-domain methods (the study of special analytic functions) and time-domain methods (the so-called Lyapunov methods based on the study of some Lyapunov functionals or functions). Since our interest concerns the stability behaviour depending or not depending on the size of delay, we have considered the Lyapunov methods as more relevant to our study. The idea [18] is to construct a special functional which is 'positive-definite', has an 'infinitesimal upper limit' and which has the derivative computed along the trajectories of the given system 'negative-definite'.

It is important to note that in the particular case we are studying, *stability* of the closed-loop neutral functional differential equations is *not sufficient* for stability of the robotic task, in the sense that there can be loss of contact between the robot's tip and the environment's surface even if the closed loop is stable.

This paper is organised as follows: In Section 2, we derive the closed-loop equations and we analyse the stability of the associated functional differential equations; Section 3 is devoted to the contact instability problem, i.e. conditions under which the interaction force has constant sign. Finally, some conclusions are given in Section 4.

Notations. $G_{[a,b)}(t)$ denotes the value of the function $G(t)$in $[a,b)$. For $t \in [b,c)$, this value is given by $G_{[a,b)}(\alpha t+\beta)$, where $\alpha = \frac{b-a}{c-b}$, $\beta = \frac{ac-b^2}{c-b}$. For $a = k\tau$, $b=(k+1)\tau$, $c=(k+2)\tau$, we get $\alpha = 1$, $\beta = -\tau$.

2. Preliminaries

2.1. Closed-Loop Equations

Let us consider the following control input:

$$U = -F_d + \lambda_1 \tilde{F}_m(t) + \lambda_2 \int_\tau^t \tilde{F}_m(z)dz \qquad (2)$$

where F_m is the *measured force*. We take the *desired interaction force* $F_d < 0$ and constant. Note that integration is taken on $[\tau,t]$ because we need an initial condition given on $[0,\tau)$ to define the control law. It makes no sense to define U in (2) without defining an initial condition on the interaction force.

When there is no delay in the force measurements (i.e. $\tau = 0$) and U is applied from 0, then (1) is equal to

$$(1+\lambda_1)\tilde{F}(t) + \lambda_2 \int_0^t \tilde{F}(z)dz = 0 \qquad (3)$$

which leads to the algebraic equation $F \equiv F_d$ if $U(0) = -F_d + \lambda_1 \tilde{F}(t)$. Following Yoshikawa [39] and MacClamroch and Wang [23], the case when $\tau \neq 0$ is very different from the case $\tau = 0$. Indeed, if $\tau=0$, the integral action is from a theoretical point of view useless since $F = F_d$ for all $t \geq 0$ and it to be considered for practical purpose only. Conversely, when non-zero delay is present, the integral term plays a significant role in the closed-loop dynamics, as we shall see in this paper.

Now let us assume that the measured force $F_m(t) = F(t-\tau)$, where $\tau > 0$ represents a strictly positive time-delay. Then (3) becomes

$$\tilde{F}(t) + \lambda_1 \tilde{F}(t-\tau) + \lambda_2 \int_0^{t-\tau} \tilde{F}(z)dz = 0 \qquad (4)$$

which can be rewritten defining $x(t) = \int_0^{t-\tau} \tilde{F}(z)dz$ as

$$\dot{x}(t) = -\lambda_1 \dot{x}(t-\tau) - \lambda_2 x(t-\tau) \qquad (5)$$

Remark 2. We assume implicitly that the robot is in contact with the environment. In particular, as long as the contact is maintained, then $q = \dot{q} = 0$. This is the reason why adding a damping term $-\lambda_2 \dot{q}$ in (2) is not necessary for the analysis.

Remark 3. Instead of the control in (2), we could have also supposed that U is given by

$$U = -F(t) + \lambda_1 \tilde{F}(t) + \lambda_2 \int_\tau^t \tilde{F}(z)dz$$

which yields

$$\dot{x}(t) = (1-\lambda_1)\dot{x}(t-\tau) - \lambda_2 x(t-\tau) \qquad (6)$$

Since both equations in (5) and (6) are quite similar, it is sufficient to study (5).

Remark 4. Let us consider the following controller:

$$U = -F_d + \lambda_1 \tilde{F}_m(t) + \lambda_2 \dot{\tilde{F}}_m(t) \qquad (7)$$

The closed-loop equation in the ideal case is

$$(1+\lambda_1)\tilde{F}(t) + \lambda_2 \dot{\tilde{F}}(t) = 0$$

which is a first-order linear differential equation. Now assume $F_m(t) = F(t-\tau)$. Then we get

$$\tilde{F}(t) + \lambda_1 \tilde{F}(t-\tau) + \lambda_2 \dot{\tilde{F}}(t-\tau) = 0$$

which can be written with $x(t) = \tilde{F}(t-\tau)$ and $\lambda_2 > 0$ as

$$\dot{x}(t) = -\frac{\lambda_1}{\lambda_2} x(t) - \frac{1}{\lambda_2} x(t+\tau)$$

which is a non-causal differential equation.

The closed-loop system is ill posed in this case, because the open-loop system in (1) is algebraic and the controller in (7) is a function of the output derivative.

2.2. Stability of the Functional Closed-Loop Equations

Using fundamental results of the stability theory for neutral functional differential equations [18] we can state the following result:

Proposition 1. Supposing $0 < \lambda_1 < 1$, $\lambda_2 > 0$ the trivial solution of the NFDE (5) is uniformly asymptotically stable for any constant delay τ satisfying

$$\tau < \frac{1 - \lambda_1}{\lambda_2} \tag{8}$$

The proof is given in the Appendix.

Remark 5. The condition $\lambda_1 < 1$ is an obvious necessary condition for stability analysis since that is equivalent to the stability of the operator $\mathcal{D}_1\phi = \phi(0) - \lambda_1\phi(-\tau)$.

Remark 6. If $\lambda_1 = 0$ in (2), then stability of (3) remains unchanged. On the contrary, (5) becomes

$$\dot{x}(t) = -\lambda_2 x(t - \tau) \tag{9}$$

which is no longer an NFDE, but an RFDE (retarded functional differential equation [21].

In this situation, the real condition of stability [4,28] is

$$\tau < \frac{3\pi}{2\lambda_2}$$

With $\lambda_1 = 0$ in (8) we obtain a 'relatively restrictive' condition

$$\tau < \frac{1}{\lambda_2}$$

which can be obtained using a Razumikhin type theorem [18,28].

Furthermore, from (8) λ_2 is a 'measure' for the admissible delay size for a given λ_1.

Remark 7. If $\lambda_2 = 0$ we obtain

$$\dot{x}(t) = -\lambda_1 \dot{x}(t - \tau) \tag{10}$$

which is a functional equation in $\dot{x}(t)$. A necessary and sufficient condition for the stability of (10) is $|\lambda_1| < 1$ [18]. U in (2) is a simple proportional force feedback in this case.

Wen and Murphy [37] have already pointed out the necessity of having $|\lambda_1| < 1$ for stability of a proportional force feedback using a different argument.

3. Contact Instability Phenomena Analysis

In the previous section we provided mathematical conditions that guarantee stability of the closed-loop system. In this section we analyse conditions under which the robot may lose the contact, and we clearly identify time-delays to be a cause of possible bouncing of the robot's tip on the environment.

It is important to note that the stability conditions derived in the foregoing section do not guarantee a constant sign of the system's state, i.e. of the interaction force. Indeed, the closed-loop system may be asymptotically stable but oscillating. In our case, a positive sign of F during a non-zero time interval means that contact is lost, and the corresponding open-loop system is not the same (namely, the equation in (1) becomes a second-order differential equation). Therefore mathematical stability conditions deduced in the previous section (which are sufficient conditions for the stability of the considered functional differential equation) are not sufficient for stability of the contact task.

3.1. Conditions for Interaction Force Constant Negative Sign

In the next two paragraphs, we analyse the sign of the interaction force F, when $\lambda_1 > 0$, $\lambda_2 = 0$ (proportional force feedback) and when $\lambda_1 > 0$, $\lambda_2 > 0$ respectively (proportional interaction force feedback).

3.1.1. Proportional Force Feedback

Let us first analyse the case when $\lambda_2 = 0$, i.e. $U = -F_d + \lambda_1\tilde{F}_m$. The closed-loop equation is given in [25]. Since we assume that the robot is in contact with the environment before the first instant (taken here to be equal to τ) when U is applied, we must assume that on $[0,\tau]$ the contact force has a certain value, say $U_{[0,\tau]} = -F_0$, $F_0 < 0$. Note that if we do not suppose that contact is established before U is applied, then we must study the transition phase between unconstrained and constrained motion [6]. For more details on the complexity analysis of the *transition phase* see, for instance, Brogliato [5].

Since we study the stability properties of the constrained-motion task, we must assume that contact has in fact always occured in the past, which is an implicit assumption in all stability analysis of hybrid force/position controllers [20,30,39]. From a mathematical point of view, the value of the interaction force F on $[0,\tau]$ is a necessary condition for the existence of a solution to the considered functional equation.

The problem can be formulated as follows: for given F_0 and F_d, find conditions such that the sign of F remains constant for all $t \geq 0$.

Since on $[0,\tau)$, we have $F_{[0,\tau)} = F_0 < 0$ and from (1) and (2) with $\lambda_2 = 0$ on $[\tau,2\tau)$

$$F_{[\tau,2\tau)} = -U_{[\tau,2\tau)} = F_d - \lambda_1 F_{[0,\tau)} + \lambda_1 F_d \quad (11)$$

Thus we obtain

$$|F_d| > \frac{\lambda_1}{1+\lambda_1}|F_0| \Leftrightarrow F_{[\tau,2\tau)} < 0 \quad (12)$$

We have the following:

Proposition 2.

$$\text{sgn}[F_{[k\tau,(k+1)\tau)}(t)] = \text{sgn}[F_{[0,\tau)}] \Leftrightarrow |F_d| > \frac{\lambda_1}{1+\lambda_1}|F_0|, \quad (13)$$

where sgn denotes the sign function.

Proof. It follows by simple computations (from (11)) that

$$\begin{aligned} F_{[2\tau,3\tau)}(t) &= -U_{[2\tau,3\tau)}(t) \\ &= F_d - \lambda_1[F_{[\tau,2\tau)}(t-\tau) - F_d] \quad (14) \\ &= (1-\lambda_1^2)F_d + \lambda_1^2 F_0 \end{aligned}$$

Since $\lambda_1^2 < 1$ (from Proposition 1), $F_d < 0$, $F_0 < 0$, we have $F_{[2\tau,3\tau)}$ (t) < 0 for all $t \in [2\tau,3\tau)$. Now (from [1])

$$\begin{aligned} F_{[3\tau,4\tau)}(t) &= -U_{[3\tau,4\tau)}(t) \\ &= F_d - \lambda_1[F_{[2\tau,3\tau)}(t-\tau) - F_d] \\ &= (1+\lambda_1^3)F_d - \lambda_1^3 F_0 \end{aligned}$$

and thus

$$F_{[3\tau,4\tau)}(t) < 0 \Leftrightarrow |F_d| > \frac{\lambda_1^3}{1+\lambda_1^3}|F_0|$$

By induction, we have for all integers $k \geq 1$

$$F_{[2k\tau,(2k+1)\tau)}(t) < 0, \qquad \forall t \in [2k\tau, (2k+1)\tau)$$

$$F_{[(2k+1)\tau,(2k+2)\tau)}(t) < 0, \qquad \forall t \in [(2k+1)\tau, (2k+2)\tau) \Leftrightarrow |F_d| > \frac{\lambda_1^{2k+1}}{1+\lambda_1^{2k+1}}|F_0|$$

Since $F_0 < 0$ and for all integers $k \geq 1$ and for all $0 < \lambda_1 < 1$

$$\frac{\lambda_1^k}{1+\lambda_1^k} \leq \frac{\lambda_1}{1+\lambda_1}$$

We finally get

$$F(t) < 0 \Leftrightarrow |F_d| > \frac{\lambda_1}{1+\lambda_1}|F_0| \qquad \square$$

3.1.2. Proportional-Integral Force Feedback

We now examine the general case when $\lambda_1 > 0$, $\lambda_2 > 0$. The closed-loop equation is given by (5).

Supposing that $U_{[0,\tau)} = -F_0 > 0$, we obtain

$$U_{[\tau,2\tau)}(t) = -F_d + \lambda_1(F_0 - F_d) + \lambda_2(t-\tau)(F_0 - F_d), \qquad \forall t \in [\tau,2\tau) \quad (15)$$

We also suppose that

$$|F_d| > \frac{\lambda_1}{1+\lambda_1}|F_0|.$$

Assume first $F_0 - F_d < 0$.

From (15) we get $U_{[\tau,2\tau)}(t) + F_d < 0$, *for all* $t \in [\tau,2\tau)$. It follows that

$$|F_d| > \frac{\lambda_1 + \lambda_2\tau}{1+\lambda_1+\lambda_2\tau}|F_0| \Rightarrow U_{[\tau,2\tau)}(t) > 0, \qquad \forall t \in [\tau,2\tau) \quad (16)$$

In this case for all $t \in [2\tau,3\tau)$ we have

$$U_{[2\tau,3\tau)}(t) = -F_d + \lambda_1[F_{[\tau,2\tau)}(t-\tau) - F_d] + \lambda_2\int_0^\tau (F_0 - F_d)dz + \lambda_2\int_\tau^{t-\tau}(F_{[\tau,2\tau)}(z) - F_d)dz \quad (17)$$

Since $F_0 - F_d < 0$ and $F_{[\tau,2\tau)}(t-\tau) - F_d > 0$ (from (15)), nothing can be straightforward concluded on the sign of $U_{[2\tau,3\tau)}(t)$.

However, note from (17) that $U_{[2\tau,3\tau)}(t)$ is a polynomial of order 2 in t having coefficients depending on λ_1, λ_2, F_d, F_0.

Notice that we have for $t \in [k\tau,(k+1)\tau)$

$$\int_\tau^t F_m(z)dz = \sum_{j=0}^{k-2}\int_{j\tau}^{(j+1)\tau} F_{[j\tau,(j+1)\tau)}(z)dz + \int_{(k-1)\tau}^{t-\tau} F_{[(k-1)\tau,k\tau)}(z)dz \quad (18)$$

Therefore, U in (1) is a polynomial of order k in t for $t \in [k\tau, (k+1)\tau)$ by simple induction from [19].

It is important to note that although $F_0 - F_d < 0$, from (15) and (16), $F_{[\tau,2\tau)}(t) - F_d > 0$ and $F_{[\tau,2\tau)}(t) < 0$. This means that there is a jump in the interaction force at $t = \tau$, but no loss of contact. One sees that it is difficult to draw such conclusions from [19]. However, note that since

$$U_{[k\tau,(k+1)\tau)}(t) = -F_d + \lambda_1(F_{[(k-1)\tau,k\tau)}(t-\tau) - F_d) + \lambda_2\int_\tau^t \tilde{F}_m(z)dz \quad (19)$$

we have the following:

Proposition 3. $U(t)$ is piecewise continuous with discontinuities at $t = j\tau$, for all integers $j \geq 1$. The jumps' magnitudes are bounded and tend towards 0 as $t \to \infty$.

Proof. From (19) it follows that $U(t)$ is continuous on $(k\tau,(k+1)\tau)$, for all integers $k \geq 1$. It follows by simple computations that

$$F_{[\tau,2\tau)}(\tau+) - F_{[0,\tau)}(\tau-) = -(1 + \lambda_1 + \lambda_2\tau)(F_0 - F_d)$$

$$F_{[2\tau,3\tau)}(2\tau_+) - F_{[\tau,2\tau)}(2\tau_-) = -\lambda_1(F_{[\tau,2\tau)}(\tau+) - (F_{[0,\tau)}(\tau_-))$$

By induction we can prove that

$$F_{[(k+1)\tau,(k+2)\tau)}((k+1)\tau_+) - F_{[k\tau,(k+1)\tau)}((k+1)\tau_-) = (-1)^k\lambda_1{}^k(F_{[\tau,2\tau)}(\tau_+) - F_{[0,\tau)}(\tau_-)) \quad (20)$$

From (20) it follows that if $F_D \neq F_0$, we have discontinuities in F(t) and $U(t)$ respectively, for all points $t{=}j\tau$, with j a positive integer.

Furthermore, from Proposition 1, $|\lambda_1| < 1$ and from [35] it follows that the jumps' magnitude tends towards 0 when $k \to \infty$, which completes the proof. □

Remark 8. From the proof of Proposition 3, a necessary and sufficient condition to have no jumps in the closed-loop system's solution is that $U_{[\tau,2\tau)}(\tau) = -F_{[0,\tau)}$, which is equivalent to having $F_{[0,\tau)} = F_d$. In general such a condition is not verified, so the interaction force possesses discontinuities. Furthermore, the sign of the jump is alternatively positive and negative.

Let us now give sufficient conditions for F in (1) and (2) (see also (19)) to have a constant negative sign for all $t \geq 0$.

Proposition 4. There exist $\lambda_1^* > 0$, $\lambda_2^* > 0$ such that for any $0 < \lambda_1 < \lambda_1^* < 1$, $0 < \lambda_2 < \lambda_2^*$, we have $\mathrm{sgn}(F(t)) = -1$ for all $t \geq 0$.

Proof. From Proposition 2, $U(t)$ and $F(t)$ have constant sign for $\lambda = 0$. Since $U(t)$ and $F(t)$ are continuously dependent on λ_1 and λ_2 on $[\tau,2\tau)$, from (13) and (16) it follows that there exists a $\lambda_2^{(1)} > 0$, such that $F(t)$ and $U(t)$ still have constant sign on $[\tau,2\tau)$.

Since $F_{[\tau,2\tau)}(2\tau_-) < 0$, there always exists a $\lambda_1^{(1)}{>}0$ such that $F_{[2\tau,3\tau)}(2\tau_+) < 0$ for any bounded $F_d{-}F_0$ (see Proposition 3).

Following a similar reasoning, if $F_{[k\tau,(k+1)\tau)}((k{+}1)\tau_-) < 0$, *there exists a* $0 < \lambda_1^{(k)} < 1$ such that $F_{[(k+1)\tau,(k+2)\tau)}((k{+}1)\tau_+) < 0$ for any bounded $F_0 - F_d$. There also exists a $\lambda_2^{(k)} > 0$ such that $F_{[(k+1)\tau,(k+2)\tau)}(t) < 0$, $t \in ((k+1)\tau, (k+2)\tau)$, from the polynomial form of $F_{[(k+1)\tau,(k+2)\tau)}(t)$ in λ_1, λ_2.

From Proposition 1, it follows that there exists a bounded maximum force magnitude F_M. Let k_M be the index corresponding to the maximum force magnitude. Imposing $F_{[k_M\tau,((k_M+1)\tau)}(t) < 0$ on $t \in [k_M\tau,(k_M{+}1)\tau)$, we can deduce the corresponding maximal values $0 < \lambda_1^* < 1$, $\lambda_2^* > 0$.

Clearly, it is difficult to find k_M explicitly, but the important fact here is about the existence of λ_1^* and λ_2^* (the stability of the NFDE associated to the closed-loop system guarantees this existence).
Finally, let us note that if $F_{[0,)\tau} - F_d > 0$, similar conclusions hold since $F_{[\tau,2\tau)}(t) - F_d < 0$ from (13), and so on. □

Remark 9. Wen and Murphy [37] show that the integral force feedback gain has to be small enough to guarantee robustness with respect to environment flexibilities, when the measured force contains time-delays. Proposition 4 shows that even in the ideal rigid case, small λ_2 improves the closed-loop behaviour, and allows larger delay (see (8)) for fixed λ_1.

3.2. Bouncing Phase Analysis

In this section we analyse the behaviour of the system when the 'robot' loses contact with the environment, which implies a phase of rebounds since we deal with the rigid case.

The point is to determine if this phase is stable or not, i.e. will the mass be stuck on the environment again and will the rebounds stop?

3.2.1. Proportional Force Feedback

Let us consider $\lambda_2 = 0$. If (12) is not satisfied, then $F_{[\tau,2\tau)}(\tau) > 0$: the closed-loop equations are such that there is a jump in the interaction force at $t = \tau$, and the control force $U(\tau)$ is negative for a non-zero time interval starting in τ. From (11) the control input on $t \in [\tau,2\tau)$ is constant and given by

$$U_{[\tau,2\tau)}(t) = -(1 + \lambda_1)F_d + \lambda_1 F_{[0,\tau)} < 0$$

In conclusion the contact is lost at $t = \tau$ and the interaction force remains positive on the whole interval $[\tau,2\tau]$.

For $t \geq 2\tau$, the measured interaction force is zero and the control input becomes constant and is given by

$$U_{[2\tau,t_f)}(t) = -(1 + \lambda_1)F_d > 0$$

Note also that (1) becomes $m\ddot{q}(t) = U$ for $t \geq \tau$. We get on $[\tau,2\tau)$

$$q(t) = \frac{U_{[\tau,2\tau)}}{2m}(t - \tau)^2 \tag{21}$$

and on $[2\tau,t_1)$

$$q(t) = \frac{2U_{[2\tau,t_f)}}{2m}(t - 2\tau)^2 + 2\frac{U_{[\tau,2\tau)}}{m}\tau \tag{22}$$
$$(t - \tau) + \frac{2U_{[\tau,2\tau)}}{m}\tau^2$$

where t_f is the final impact time and t_1 is the time of the first impact between the mass and the environment.

Let us simply note that since the input force applied to the mass is constant for $t \geq 2\tau$, the system is strictly identical to a bouncing ball submitted to gravity.

Then a necessary and sufficient conditon to get $t_f < \infty$ is that the restitution coefficient $e > 0$ between the mass and the environment be <1; see, for example, Wang [36].

We suppose that $t_f < \infty$. Roughly, a relation similar to [15] is true between two impacts, the physical impact law providing initial conditions for $\dot{q}(t)$ after each impact (note that $q(t_k) = 0$ at impact times t_k).

On $t \in [t_f,t_f + \tau)$, we get

$$U_{[t_f,t_f+\tau)}(t) = -(1 + \lambda_1)F_d > 0$$

and on $[t_f, t_f + 2\tau)$, we get

$$U_{[t_f+\tau,t_f+2\tau)}(t) = -(1 + \lambda_1)F_d \tag{23}$$
$$+ \lambda_1 F_{[t_f,t_f+\tau)}(t - \tau) = -(1 - \lambda_1^2)F_d$$

which is positive since $0 < \lambda_1 < 1$.

By induction, one can show that

$$U_{[t_f+k\tau,t_f+(k+1)\tau)}(t) = \tag{24}$$
$$-(1 + (-1)^{k+1}\lambda_1^{k+1})F_d > 0$$

Thus, there is no loss of contact any longer.

In conclusion, we have proved the following:

Proposition 5. Assume $\lambda_2 = 0$, and that (12) is not satisfied. Then there is at most one detachment of the mass from the constraint surface $q = 0$. Once contact is remade, after a series of rebounds, the interaction force remains with constant negative sign.

Remark 10. The condition (12) is fulfilled if $F_0 = (1 + \lambda_1)F_d$. We could have supposed that $U_{[0,\tau)} = -(1 + \lambda_1)F_d$ and that the force feedback is switched on at $t = \tau$. Note, however, that the analysis done in the preceding subsection is only true if there is no force disturbance acting on the system.

Assume that a disturbance $F_p = F_{M\chi\Delta}$ acts on the system on $t \in \Delta$, where $\chi\Delta$ is the characteristic function of the interval $\Delta \subseteq [k\tau, (k + 1)\tau)$ for some $k > 0$. The stability condition in (12) becomes

$$|F_d| > \frac{\lambda_1}{1 + \lambda_1}\left|F_{[(k-1)\tau,\kappa\tau)} - \frac{F_M}{\lambda_1}\right|$$

If $F_{[(k-1)\tau,k\tau)} - \frac{F_M}{\lambda_1} < 0$, and if $F_M < 0$, then $|F_M| < \lambda_1|F_{[(k-1)\tau,k\tau}|$: note that if this condition is not satisfied, then the robot will lose contact on $[k\tau, (k+1)\tau)$.

Furthermore, the condition $|F_M| \leq |F_{[(k-1)\tau,k\tau)}|$ is necessary and sufficient for the robot not to lose the contact on $[(k\text{-}1)\tau,k\tau)$, but it is not sufficient for contact stability for $t \geq k\tau$. The stability analysis follows the same lines with F_0 replaced by $F_{[(k-1)\tau,k\tau)} - \frac{F_M}{\lambda_1}$.

Remark 11. Following Remark 1, let us note that clearly the impact for transition phase should behave better if the input has a damping term. This, however, does not modify our analysis as long as the velocity $\dot{q}(t)$ feedback contains no time delay.

3.2.2. *Proportional-Integral Force Feedback*

Let us analyse the case $\lambda_2 > 0$. For the sake of simplicity, we suppose that (16) is not verified, so that $U_{[\tau,2\tau)}(t) \leq 0$ for $t \in [t_1,2\tau)$; i.e. there is loss of contact at $t = t_1$. Since $U_{[\tau,2\tau)}(t)$ is still given by (15), we get

$$U_{[2\tau,t_1+\tau)}(t) = -F_{d[2\tau,t_1+\tau)} + \lambda_1(F_{[\tau,t_1)}(t - \tau)$$
$$- F_{d[2\tau,t_1+\tau)}) + \lambda_2\int_0^\tau (F_{[0,\tau)} \tag{25}$$
$$- F_d)dz + \lambda_2\int_\tau^{t-\tau}(F_{[\tau,t_1)}(z) - F_{d[\tau,t_1)})dz$$

where we emphasise that F_d may be time-varying by denoting its value on I as F_{dI}; for an interval I, this can be done since F_d is chosen by the designer.

We suppose that as soon as the measured interaction force $F_m(t)$ is zero, then we set $F_d \equiv 0$, i.e. on

$[t_l+\tau, t_f+\tau)$, $F_m(t) \equiv 0$ and $F_{d[t_l+\tau, t_f+\tau)} \equiv 0$, where t_f is the instant when the contact is remade after a possible series of rebounds.

For the mass to collide with the environment, it is necessary that $U_{[t_l+\tau,3\tau)}(t)$ be positive. We have

$$U_{[t_l+\tau,3\tau)}(t) = - F_{d[t_l+\tau,3\tau)} + \lambda_1(F_{[t_l+\tau,t_l+2\tau)}(t-\tau) - F_{d[t_l+\tau,3\tau)}) + \lambda_2 \int_0^\tau (F_{[0,\tau)} - F_d)dz + \lambda_2 \int_\tau^{t_l} (F_{[\tau,t_l)} - F_{d[\tau,t_l)})dz$$

so it is necessary that

$$\int_0^\tau (F_{[0,\tau)} - F_d)dz + \int_\tau^{t_l} (F_{[\tau,t_l)}(z) - F_{d[\tau,t_l)})\, dz > 0 \quad (26)$$

It should be noted that this control strategy is related to the fact that $\tau > 0$.

If $\tau = 0$, there is in the ideal case no reason for the robot to take off the surface. We do not analyse here the stability of a complete robotic task involving contact and non-contact phases: in this case the control strategy must be adapted suitably to guarantee stability.

This is outside the scope of the present work.

From the fact that after $t_l + \tau$, the control input remains constant until $t_f + \tau$, we get $U_{[t_l+\tau,3\tau)} = U_{[3\tau,t_f+\tau)}$. Thus, (26) is equivalent to

$$-t_l F_d + \tau F_0 + \int_\tau^{t_l} [(F_d - \lambda_1(F_0 - F_d) + \lambda_2 (z-\tau)(F_0 - F_d)]dz > 0$$

that is

$$(F_0 - F_d)(\tau - \lambda_1(t_l - \tau) + \frac{\lambda_2}{2} (t_l - \tau)^2) > 0 \Leftrightarrow \tau - \lambda_1(t_l - \tau) + \frac{\lambda_2}{2} (t_l - \tau)^2 < 0 \quad (27)$$

Note that from (15) we have

$$t_l = \frac{F_d - \lambda_1(F_0 - F_d)}{\lambda_2(F_0 - F_d)} + \tau \quad (28)$$

Therefore, (27) becomes

$$\lambda_2\tau - \lambda_1 \frac{F_d - \lambda_1(F_0 - F_d)}{F_0 - F_d} + \frac{1}{2}\left(\frac{F_d - \lambda_1(F_0 - F_d)}{F_0 - F_d}\right)^2 < 0 \quad (29)$$

Note that from (11)

$$w = \frac{F_d - \lambda_1(F_0 - F_d)}{F_0 - F_d} > 0$$

we deduce that if

$$- \lambda_1 w + \frac{1}{2} w^2 < 0 \Leftrightarrow \frac{\lambda_1}{1+\lambda_1} < \frac{|F_d|}{|F_0|} < \frac{3\lambda_1}{1+3\lambda_1}$$

then there exists $\lambda_2^* > 0$ such that for $0 \leq \lambda_2 \leq \lambda_2^*$, (29) is satisfied, which implies that (26) is satisfied so that $U_{[t_l+\tau,3\tau)} > 0$: λ_2^* is a function of τ and of $\frac{|F_d|}{|F_0|}$.

Contact is remade at $t = t_f$. On $[t_f,t_f+\tau)$ we still have $F_m \equiv 0$, and $F_{d[t_f,t_f+\tau)} = 0$ as well. Hence,

$$U_{[t_f+\tau,t_f+2\tau)} = -F_d + \lambda_1(-U_{[3\tau,t_f)} - F_d) + \lambda_2 \int_0^\tau (F_0 - F_d)dz + \lambda_2 \int_\tau^{t_l} (F_{[\tau,t_l)}(z) - F_d)dz + \lambda_2 \int_{t_f+\tau}^{t} (-U_{[3\tau,t_f)}(z-\tau) - F_d)dz \quad (30)$$

where we denote for simplicity $F_{d[t_f+\tau,t_f+2\tau)}$ as $F_d < 0$.

Defining

$$K = \tau - \lambda_1(t_l - \tau) + \frac{\lambda_2}{2} (t_l - \tau)^2$$

from (30) we get

$$U_{[t_f+\tau,t_f+2\tau)}(t) = -F_d(1 + \lambda_1 + \lambda_2(t - t_f - \tau)) + \lambda_2(\lambda_1 + 1)K(F_d - F_0) + \lambda_2^2(t - t_f - \tau)K(F_d - F_0)$$

Then

$$U_{[t_f+\tau,t_f+2\tau)}(t) > 0 \Leftrightarrow \frac{|F_d|}{|F_0|} > \frac{\lambda_2|K|}{1 - \lambda_2 K} \quad (31)$$

From (29), condition (31) involves F_d, F_0, λ_1, λ_2 but explicit conditions on those parameters to guarantee that (31) is verified are difficult to be carried out.

The addition of an integral term makes the system significantly more difficult to analyse than when $\lambda_2 \equiv 0$. The general tendency is that λ_2 must be small enough to obtain a 'stable' scheme.

Remark 12. We have analysed each control law separately, i.e. as if only a proportional feedback or a proportional-integral feedback was applied all the time.

We could also have supposed that both controllers are switched along a certain strategy, e.g. apply a proportional control during impact phases and integration during contact (Note that setting $F_d \equiv 0$ whenever $F_m \equiv 0$ is different from using proportional feedback since (i) both controllers need not necessarily have either the same λ_1 gain or desired force, (ii) when a switch is applied, one may assume that the integrator is initialised.)

Usually the integral feedback is applied when contact is established. Thus, the initial condition for the contact phase will depend on the proportional feedback value on the time instant when the switch is applied: the ideal switching time is t_f; in practice the controllers will be switched before or after t_f due to bad timing in the switching strategy: this reveals the complex behaviour of such systems and the role played by both the low-level part (differential equations, delay in the force control loop) and the high-level part (the strategy that schedules the switches between several controllers, bad timing). These problems have already received some attention [6,25].

4. Conclusions

In this paper we have focused on the problem of stability of a robotic manipulator submitted to holonomic constraints. An important point is that the constraints are supposed to be unilateral, in the sense that if the interaction force has the wrong sign, the contact is lost. Furthermore, we have assumed that the force feedback loop contains time delays.

Sufficient delay-dependent conditions are derived to guarantee that the robot's tip remains in contact with the surface. These conditions are found by taking into account the fact that due to the unilateral nature of the constraints the interaction force must have constant sign during the whole task. We analyse the cases of proportional and proportional-integral force feedback. To the best of the authors' knowledge, this problem has been pointed out in Wen and Murphy [37] and in Wilfinger et al. [38], but had not been treated before in the robotics and control literature.

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Appendix: Proof of Proposition 1

Let us consider the operator D: $\mathscr{C}([-\tau,0],\mathbf{R}) \rightarrow \mathbf{R}$ defined by

$$D\phi = \phi(0) - \lambda_1\phi(-\tau) - \lambda_2 \int_{t-\tau}^{t} \phi(\theta)\mathrm{d}\theta$$

where $\mathscr{C}([-\tau,0],\mathbf{R})$ is the Banach space of continuous function mapping $[-\tau,0]$ in $\mathbf{R}$.

After some algebraic manipulation the NFDE (5) can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t} Dx_t = -\lambda_2 x(t) \qquad (32)$$

where $x_t:[-\tau,0] \rightarrow \mathbf{R}$, $\mathbf{x}_t(\mathbf{t}+\theta)=\mathrm{x}(\mathrm{t}+\theta)$.

It is clear that the operator D is stable if

$$|\lambda_1| + \lambda_2 \int_{t-\tau}^{t} \mathrm{d}\theta < 1$$

or equivalently

$$|\lambda_1| + \lambda_2\tau < 1 \qquad (33)$$

which is satisfied from the Proposition statement.

In the sequel we will prove that the trivial solution of the NFDE (5) is uniformly asymptotically stable using Lyapunov's second method [18].

Inspired by Kolmanovskii and Nosov [20], we introduce the following Lyapunov–Krasovskii candidate:

$$V(x_t) = \left[x(t) - \lambda_1 x(t-\tau) - \lambda_2 \int_{t-\tau}^{t} x(\theta)\mathrm{d}\theta\right]^2 + \lambda_2 \int_{t-\tau}^{t} \left[\lambda_1 x^2(\theta) + \lambda_2 \int_{\theta}^{t} x^2(\mu)\mathrm{d}\mu\right]\mathrm{d}\theta \qquad (34)$$

We have

$$(Dx_t)^2 \leq V(x_t) \leq m(\sup_{\theta\in[-\tau,0]} |x(t+\theta)|)^2 \qquad (35)$$

where $m = 4 + 4(1 - \lambda_1)^2 + \frac{5}{2}\lambda^2{+}_2\tau^2 + \lambda_2\tau$. Inequalities (35) mean that the Lyapunov candidate

V is positive-definite and has an infinitesimal upper bound.

Using the inequality

$$2\lambda_2 x(t)\int_{t-\tau}^{t} x(\theta)\mathrm{d}\theta \leq \lambda_2\left(\tau x^2(t) + \int_{t-\tau}^{t} x^2(\theta)\mathrm{d}\theta\right)$$

we have

$$\begin{aligned}\dot{V}(x_t) &= -\lambda_2(x(t) - \lambda_1 x(t-\tau))^2 \\ &\quad - 2(\lambda_2 - \lambda_1\lambda_2 - \lambda_2\tau)x^2(t) \\ &\leq -2\lambda_2(1 - \lambda_1 - \lambda_2\tau)x^2(t)\end{aligned} \tag{36}$$

Based on the Proposition hypothesis, it follows that the trivial solution of (5) is uniformly asymptotically stable (from (35) and (36) see Theorem 8.1, p. 293 in Hale and Lunel [18]).

□

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Discussion on: 'Force Measurement Time-Delays and Contact Instability Phenomenon' by S.-I. Niculescu and B. Brogliato

Coordinated by A. Besançon-Voda

1. Discussion by A. Tornambè[1]

1.1. Introduction

In the paper under discussion, the authors, S.-I. Niculescu and B. Brogliato, study one of the most important problems that can arise when two or more mechanical systems interact with each other or with the external environment: the possibility that the contact between parts of the mechanical systems and of the external environment (which is assumed to be maintained in many applications, such as force control) is lost during some particular tasks. They study such a problem with reference to a simple mechanical system: a mass, with one degree of freedom, interacting with a fixed obstacle, under the action of two very simple (but widely used in practical applications) control laws, namely a proportional standard regulator, and a proportional–integral standard regulator. Such control laws can be proven to assure a sort of asymptotic stability for the closed-loop system, when no time-delay is present in the feedback loop (e.g., by means of the technique given in Tornambè [1]). The authors, in this paper, show that if a time-delay is present in the feedback loop then some instability phenomena can be created, such as a limit cycle yielding the loss of contact between the mass and the obstacle. In particular, they give sufficient conditions to ensure that the contact is not lost during the task.

In this discussion, we would like to improve the study reported by the authors with some experimental results, which can validate the analysis they have made. For the sake of simplicity, the experimental results have been restricted to the case of the proportional standard regulator, but it seems that a similar behaviour is also obtained when the second type of controller is used.

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1.2. Experimental Results

The experimental test has been carried out on a very simple prototype available at the Automatic Control Laboratory of the Università di Roma Tre. The prototype is constituted by a single-link robot, which can interact with a fixed obstacle located in the working space. The link (of length 0.16 m) and the obstacle are made by steel: at the point of contact of the link with the obstacle, there is a simple device, constituted by a small and flexible element, with known elastic characteristics, whose deformation, obtained through a mechanical transmission by an optical encoder with high resolution, allows the measurement of the exchanged force to be reconstructed. The link is driven by a DC motor (Pittman, GM9413D627), with a voltage range of 12 V. Since the motor is a high-speed, relatively low-torque actuator, it is geared down by a speed reducer, connecting the motor rotor and the link. The employed reducer, for which negligible backlash is ensured, has a gear ratio of 127.7:1. A unique power unit is used for the entire system, containing in particular: (i) the servo amplifier to drive the DC motor, (ii) the digital-to-analogue converter, and (iii) the counter for the software decoding of the encoder output. The system is interfaced with a Pentium MMX, 200 MHz personal computer by a Keithly digital input/output board. The control and the DSP software are both written in C++. The sampling time used for the experiment is 5 ms, and the duration of the entire experiment is 4 s. For safety reasons and to prevent damage to the system, the power unit has been designed so that the voltage command supplied to the motor is saturated to 12 V; this saturation does not limit the experiment, as we will see from the experimental results.

The experimental test was carried out in the following way. The link starts in contact with the obstacle, with zero interaction force; the aim of the proportional controller is then to regulate the exchanged force to a level of 0.5 N (this value is compatible with the force that can be generated by the DC motor). The experiment was repeated three times: (i) in the first test, the exchanged force was used for the computation of the voltage command with no time-delay; (ii) in the second test, there was a time-delay of 0.5 s; and (iii) in the third test, there was a time-delay of 1 s. Figure 1 reports the time histories of the exchanged forces in the three cases, whereas Fig. 2 reports the relative voltage inputs to the DC motor. As can be seen from these figures, when there is no time-delay the mechanical system shows a convergent behaviour: in particular, the exchanged force converges to a value of about 0.3 N, which is smaller than 0.5 N, as the gain employed in the proportional controller has been

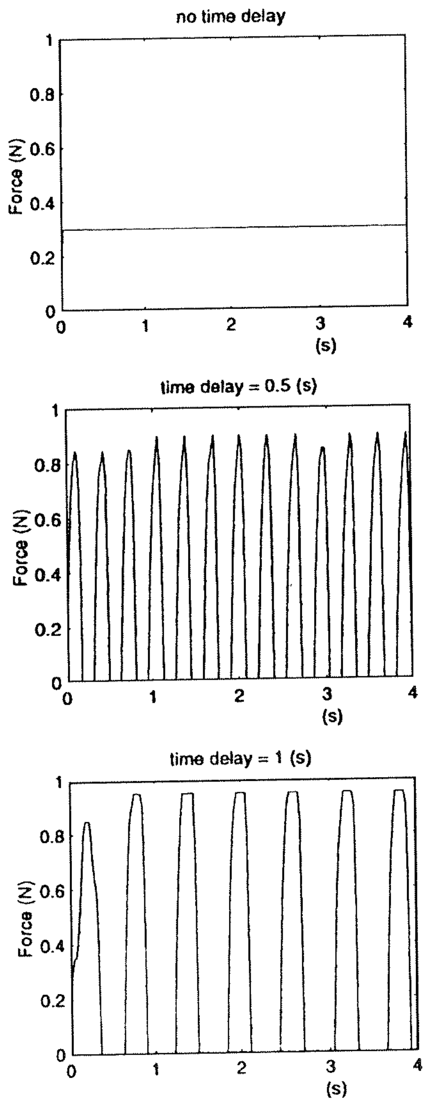

Fig. 1. Time histories of the contact force.

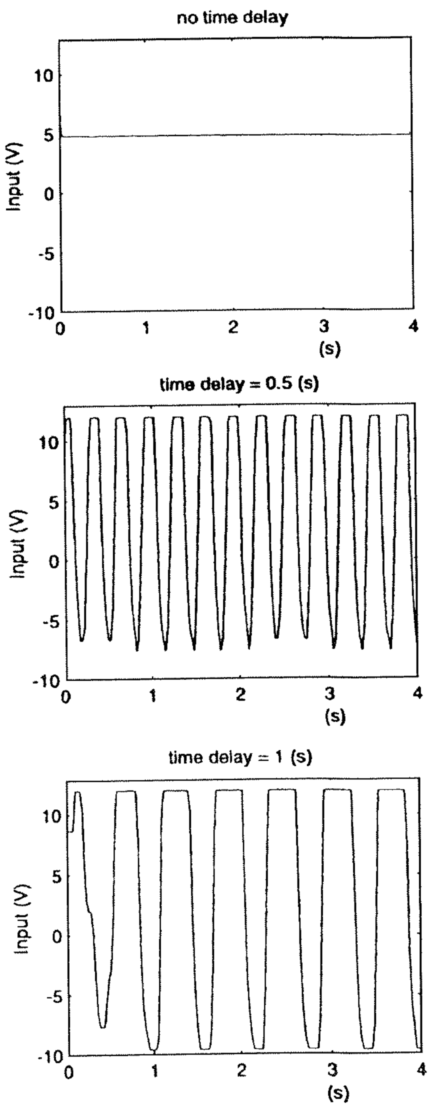

Fig. 2. Time histories of the input voltage.

chosen sufficiently small; with an increase of the value of the gain, the steady state error can be reduced as far as this gain is compatible with the voltage saturation of the DC motor. When there is a time-delay of 0.5 s in the feedback loop, the system shows an unstable behaviour: in particular, we have a limit cycle which implies an oscillating behaviour for the exchanged force; each time the exchanged force tends to be negative, the contact between the link and the obstacle is lost (in 4 s, the contact is lost 13 times). When there is a time-delay of 1 s in the feedback loop, the system shows a similar unstable behaviour: in particular, we have a limit cycle which implies an oscillating behaviour for the exchanged force slower than the other case; each time the exchanged force tends to be negative, the contact between the link and the obstacle is lost (in 4 s, the contact is lost seven times). As can be seen from the last two tests, in both cases we have different periods of oscillations, whereas the magnitude of the oscillation seems to be the same: this latter behaviour should be due to the saturation of the input voltage of the motor, as can be recognised from Fig. 2, where the input voltages applied in the three cases are reported.

This behaviour seems to be in perfect agreement with the theoretical analysis made by the authors of the paper under discussion, and can be considered as a partial experimental validation of their work.

2. Final Comments by the Authors S.-I. Niculescu[2] and B. Brogliato[3]

We would like to thank Professor Tornambè very much for having significantly improved our result by performing experiments that show that delays in the force measurements play a major role in the possible detachments of the robot's tip from the unilateral constraint. Besides its practical relevance this simple case study is one of the very few physical examples that yields a neutral differential-equation in conjunction with non-smooth effects due to impacts and unilateral constraints.

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