Dynamic transition simulation of a walking anthropomorphic robot

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Abstract

This paper deals with an approach to carry out the transitions of a walking robot. A set of elementary transformations is proposed to modify the locomotion parameters allowing to obtain various kinds of walks. Some simulation results are finally given.

1 Introduction

The problem of dynamic locomotion and gait generation for biped robots has been studied theoretically and experimentally by many researchers with different approaches. Mc Geer built simple mechanical devices to demonstrate the use of passive dynamics where natural gaits are generated by passive interaction of gravity, inertia and friction with ground [1]. Based on the idea of Mc Geer, Goswami and Espiau have theoretically studied dynamics of compass device and its limit cycles and showed the passive chaotic walk of the compass on an inclined surface [2]. Raibert and al have developed passive hopping and running machines, which require little energy expenditure and simplified control [3]. Hodgins describes algorithms for generating simple biped run-to-walk and walk-to-run transitions for a simple biped device [4]. Forman’s work shows the weak energy consumption of impulsive control to generate ballistic motions of an anthropomorphic biped [5]. Grishin divides the step cycle into time intervals to design the periodic nominal regime of two experimental biped systems. Kinematic variables were computed as time polynomials of the fifth or fourth order. The choice of the boundary values permits to determine the coefficients of all polynomials [6]. Kajita and Tani use the linear inverted pendulum model to control biped walking on a rugged terrain by assuming an ideal model with massless legs, which moves on a straight line and rotates at a constant angular velocity [7]. Searchers of Waseda University have developed several anthropomorphic robots able to walk dynamically by using control method based on the ZMP (zero-moment-point) [8]. The approach of ZMP was theoretically studied and established by Vukobratovic [9].

The problem of motion generation for biped walking robots has been the subject of much research. However, the transition from a slowly walking mode to a faster one (and the reverse process) for an anthropomorphic robot has not been yet studied.

This paper presents a method to produce various types of walking bipeds gaits and transitions based on elementary transformations of a reference locomotion cycle. In the second section, we exhibit the approach of reference cycle and the way to reproduce it. In the third section, we propose a set of elementary transformations to modify the reference cycle. Furthermore, the method to generate a transition from a given walking mode to another is described. In the fourth section, some results are given concerning the transition from slowly to faster walking and the inverse process for different walking modes. Some conclusions and further developments are exposed in the last section.

2 Reference gait generation

Our aim is to carry out a stable nominal walking regime for an anthropomorphic biped model. In order to reach several stable cycles as near as possible of human walking regimes, we have to establish a parametric formulation of the locomotion cycle. This parametric formulation is based on biomechanical studies which give on one hand the average geometry and mass distribution of a standard human and on the other hand statistical data of all joint angles during a steady walk on a horizontal plane.

It should be noted that the time evolution of the ankles positions \( (x_j(t), y_j(t)) \) relatively to the body frame \( R_A \) (fig. 1) is the major factor on producing the velocity of the structure in a plane defined by gravity and direction of displacement. Indeed, the time evolution of these parameters fixes the frequency, the time delay between the two legs and the step lengths of the walking device.

Using the forward kinematics of a leg, these positions are computed as functions of the hip and knee angles values \((\theta_{\text{hip}}, \theta_{\text{knee}})\) \(j=1,2\).
The reference walking cycle $y_j = f(x_j)$ of the ankle position relatively to the body is then obtained (fig. 2). The set of variables which give the positions and orientations of the two ankles is $\text{Var} = \{x_j, y_j, q_j\}_{j=1,2}$.

In order to reproduce the biped global motion, we have to approximate the recorded time evolution of these parameters satisfying the following conditions:

(i) the local extremas, which fix the motion amplitudes, have to be kept $\forall u \in \text{Var}$.

(ii) the acceleration for each variable has to be continuous.

(iii) the maximal jerk of each variable has to be minimized.

To achieve this aim, we use time polynomials functions of the 5th order to approximate each variable $u \in \text{Var} [6]$. The locomotion cycle is divided into several time intervals for each variable: $T_u = \bigcup_{k=1}^{n_u} [t_{uk}, t_{uk+1}]$, where $T_u$ is the cycle duration and $n_u$ the number of intervals for the $u$ variable in set $\text{Var}$. Coefficients of the polynomials can be thus computed as analytical functions of the boundary values:

$$\forall u \in \text{Var}, 1 \leq k \leq n_u, \exists! P_5(t)/$$

$$P_5(t_k) = p_d \quad P_5(t_{k+1}) = p_f$$

$$P_5(t_k) = v_d \quad P_5(t_{k+1}) = v_f$$

$$P_5(t_k) = a_d \quad P_5(t_{k+1}) = a_f$$

where $(p_d, p_f), (v_d, v_f)$ and $(a_d, a_f)$ are respectively the boundary conditions for positions, velocities and accelerations on the limits of each interval.

The general form of the polynomial function $P_5$ is the following:

$$P_5(t) = \sum_{i=0}^{5} A_i t^i \text{ with } A_i = f_i(p_d, p_f, v_d, v_f, a_d, a_f)$$

An iterative procedure permits us to choose good boundary conditions for acceleration minimizing the maximal jerk. As an example, the results obtained for the ankle acceleration joint is given with different time polynomials functions: 3rd order, 5th order with 0-acceleration boundary conditions, 5th order with near optimizing boundary conditions (fig. 3).

By using cond (i) points A,B,D,E,F of the locomotion cycle are obtained (fig. 2). However, a set of tests have been carried out showing that these points are not sufficient to obtain a good approximation of the locomotion cycle. For symmetry reasons, we choose points that occur at instant $t_i$ and for which $y_j(t_i) = y_j(t_k)$ with $\dot{x}_j(t_k) = 0$. This gives points $A'$ and $D'$. The locomotion cycle approximation becomes thus accurate. Thanks to the inverse dynamic model and by using a multibody dynamic software package called SDS 1, it is possible to simulate the walk of the biped robot for a given regime. This allows to analyse the nominal regime by observing the necessary joint torques, the contact forces and the body angular and linear acceleration. The inverse dynamic model requires joints positions, velocities and accelerations (ankle, knee, hip, shoulder and elbow). As it was shown, the polynomial approximation gives the positions $x_{g_j}(t) = \{x_j(t), y_j(t)\}$ and orientations of the ankles $\{q_{j}(t)\}$ and their derivatives $\{\dot{q}_{j}(t), \ddot{q}_{j}(t)\}$.

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1 SDS is a trademark of Solid Dynamics
\{ \ddot{q}_{j}(t), \dddot{q}_{j}(t) \} \text{ for the two legs } (j = 1, 2). \text{ Hip and knee joint positions } q_{j}(t) = \{q_{\text{hip}}(t), q_{\text{knee}}(t)\} \text{ and their derivatives } \dot{q}_{j}(t), \ddot{q}_{j}(t) (j = 1, 2) \text{ are obtained by using the inverse explicit kinematics models of the structure (with two rotary joints) for the two legs } (j = 1, 2):

\begin{align*}
F^{-1} &\{x_{j}(t), \dot{x}_{j}(t)\} = F_{j}^{-1}(x_{j}(t), \dot{x}_{j}(t)) \\
&\{y_{j}(t), \dot{y}_{j}(t)\} = G_{j}^{1}(\dot{x}_{j}(t), \ddot{q}_{j}(t)) \\
&\{z_{j}(t), \dot{z}_{j}(t)\} = G_{j}^{2}(\dot{x}_{j}(t), \ddot{q}_{j}(t))
\end{align*}

\(F^{-1}\) gives the relation between joint positions of the knee and hip, and cartesian positions of the ankle relatively to the body reference \(R_{A}\). \(G_{j}^{1}\) and \(G_{j}^{2}\) are the explicit inverse kinematic models of the first and second order. For the shoulder and the elbow, we can use the same process as for the hip and the knee but some biomechanical observations of the walk indicate that motions of the arms are synchronized with the motions of the legs [10]. So, we choose the following law for the shoulder joint positions:

\[q_{\text{sh}}(t) = q_{\text{sh}}^{0} + K_{s}q_{\text{sh, hip}}(t)\]

where \(q_{\text{sh}}^{0}\) is an offset depending on the inclination of the trunk relatively to the ground, \(q_{\text{sh, hip}}\) is the angle of the opposed hip and \(K_{s}\) a scaling factor.

The elbow is considered as a passive joint with a rotary spring-damper. This leads to obtain the joint torque \(\tau_{\text{elb}}\):

\[\tau_{\text{elb}}(t) = k[q_{\text{elb}}^{0} - q_{\text{elb}}(t)] - \mu q_{\text{elb}}(t)\]

where \(k\) and \(\mu\) are spring and damper coefficients, \(q_{\text{elb}}^{0}\) is an equilibrium position of the elbow.

The foot/gound interaction is introduced in the inverse dynamic model as a compliant and distributed model [11]. This model is based on a set of spring-damper elements which permit to avoid the closed loops considerations and the limited assumption of a single contact point geometrically invariant during the contact time.

In the next section, we propose a set of elementary transformations, which permit to modify the locomotion cycle and to obtain an infinite number of various walking types allowing us to carry out the transitions simulations.

3 Transition between two walking modes

The proposed transformations of the locomotion cycle are based on the assumption that the motions obtained for a given walking regime for a given structure can be extended to others new regimes. These transformations concern the spatial and temporal variations of the locomotion parameters.

3.1 Spatial transformations

a) Translation of the locomotion cycle \((T)\): The locomotion cycle is translated according to the body frame \(R_{A}\) (fig. 1) by using a simple transformation which gives the new ankle positions \((j = 1, 2)\):

\[T : \begin{pmatrix} x_{j}(t) \\ y_{j}(t) \end{pmatrix} = \begin{pmatrix} x_{j}(t) - T_{x} \cdot x_{j}(t) \\ y_{j}(t) - T_{y} \cdot y_{j}(t) \end{pmatrix}\]

where \(T_{x}\) and \(T_{y}\) are the translation parameters. Several translations of the cycle are given in fig. 4.

![Figure 4: Translation of the walking cycle.](image)

b) Rotation \((R)\): In this case, a rotation of the cycle according to the body frame origin \(A\) (fig. 1) is obtained by the following relation for each leg :

\[R : \begin{pmatrix} x_{j}(t) \\ y_{j}(t) \end{pmatrix} = \begin{pmatrix} c_{\beta_{j}} & s_{\beta_{j}} \\ -s_{\beta_{j}} & c_{\beta_{j}} \end{pmatrix} \begin{pmatrix} x_{j}(t) \\ y_{j}(t) \end{pmatrix}\]

where \(\beta_{j}\) is the rotation angle around the vector \(Z_{A}\).

c) Compression and Extension: The locomotion cycle is characterized by a reference height \(H_{\text{ref}}\) and a reference length \(L_{\text{ref}}\) of the step given by the following relations (fig. 2):

\[
\begin{align*}
L_{\text{ref}} &= x_{A} - x_{D} \\
H_{\text{ref}} &= y_{E} - y_{B}
\end{align*}
\]

Changing the step length \(L\) or/and height \(H\) leads to a new set of boundary conditions for the ankle positions \((x_{j}(t), y_{j}(t))\). This transformation can be defined as a homothetic one according to a reference point called cyclo-center \(C\) of the locomotion cycle. The cyclo-center coordinates \((x_{C}, y_{C})\) in the body frame \(R_{A}\), can be chosen anywhere until the resulting cycle is still in leg reachable workspace. The length transformation of the boundary conditions are the following :

\[
S_{L} : \begin{align*}
x_{j}^{(n)} &= (x_{C}^{(n)} - x_{C}) L_{\text{ref}} + x_{C} \\
\dot{x}_{j}^{(n)} &= \dot{x}_{C} L_{\text{ref}} \\
\ddot{x}_{j}^{(n)} &= \ddot{x}_{C} L_{\text{ref}} \\
\dddot{x}_{j}^{(n)} &= \dddot{x}_{C} L_{\text{ref}}
\end{align*}
\]

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The height transformation $S_H$ is obtained by replacing in the previous relations $x$ by $y$, $I$ by $H$ and $L_{ref}$ by $H_{ref}$.

Some examples of these transformations are given for a cyclo-center placed on $B$ (bottom of the trajectory (fig. 2)) (fig. 5).

Figure 5: Compression-Extension of the length and height of the walking cycle centered on point $B$.

### 3.2 Temporal transformations

**a) Time Coordination between legs ($D$)**: For the nominal walking regime, the motions of the two legs can be considered as the same except a time delay. A large set of coordinations between the legs can be obtained by changing this offset. The delay-transformation $D$ is given (for $j = 1, 2$) by:

$$
D:\begin{cases}
  x_j^0(t) = x_j(t - t_{j,ref}) \\
  \dot{x}_j^0(t) = \dot{x}_j(t - t_{j,ref}) \\
  \ddot{x}_j^0(t) = \ddot{x}_j(t - t_{j,ref})
\end{cases}
$$

where $t_{j,ref}$ is the time delay for the variable $x_j$. The same transformations are applied to $y_j$ and $q_j$. Several time delay can be used. For instance, we can observe a limping phenomenon by choosing an asymmetrical coordination.

**b) Compression and Extension ($C$)**: It is possible to accelerate or decelerate all the motions of the walking robot by changing the cycle duration $T_c$. Let $x_j^{(1)}(t), y_j^{(1)}(t), q_j^{(1)}(t)$ ($j = 1, 2$) be the ankle parameters for a unitary reference cycle (duration of 1 s). The new parameters of the cycle (duration $T_c$) are obtained (for $j = 1, 2$) by the relations:

$$
C:\begin{cases}
  x_j^0(t) = x_j^{(1)}(\frac{t}{T_j}) \\
  \dot{x}_j^0(t) = \frac{1}{T_j} x_j^{(1)}(\frac{t}{T_j}) \\
  \ddot{x}_j^0(t) = \frac{1}{T_j^2} x_j^{(1)}(\frac{t}{T_j})
\end{cases}
$$

The same transformations are applied to $y_j$ and $q_j$.

### 3.3 Transition

In order to carry out transitions, we have to compose the previous defined elementary transformations $(T, R, S_L, S_H, D, C)$ in the following way:

$$
\begin{pmatrix}
  x_j^0(t) \\
  y_j^0(t)
\end{pmatrix} = C_dD_dT_dR \begin{pmatrix}
  x_j(t) \\
  y_j(t)
\end{pmatrix}
$$

The $S_L$ and $S_H$ transformations are used in the computation of the boundary conditions.

The composition of elementary transformations allows us to determine the minimal set of locomotion parameters which control the locomotion process. This set is the following:

$$P = \{T_c, \beta, t_{\phi_0}, t_{\phi_1}, t_{\phi_2}, H, L, T_x, T_y, x_j, y_j\}_{j=1, 2}$$

All the previous parameters can be changed during the simulation as functions of the desired behaviour of the biped robot.

For instance, if we want to do a transition between a stable walking regime to another, we choose, first of all, by an iterative analysis, the good parameters for the first regime, and then for the second regime. In order to simulate the transition between the two walking modes, we can fix a set of parameters before an instant $t_1$, a set of parameters after an instant $t_2$, and ensure the transition during the interval $[t_1, t_2]$ by a simple linear interpolation for each variable $p_i \in P$.

### 4 Results

We give here some results obtained with our set of elementary transformations. Three applications examples are shown. The duration of the three simulations is 15 s. Acceleration and quick acceleration of the walk can be obtained with the operators $\{C, R, D\}$. Deceleration is carried out with the operators $\{R, S_L, S_H, T, D\}$.

#### 4.1 Acceleration

The first application is an acceleration of the walking process with a smooth variation of the locomotion parameters $T_c$, $\beta$ and $t_{\phi}$. During the simulation the time coordination are the following:

$$
\begin{align*}
  t_{\phi_0}^1 &= t_{\phi_1}^1 = t_{\phi_2}^1 = 0 \\
  t_{\phi_0}^2 &= t_{\phi_1}^2 = t_{\phi_2}^2 = T_c^2 \\
  T_c &= 1.15s \\
  \beta &= 0.2 rad; \text{ after transition } t > t_2 = 9.4s: T_c = 0.9s \\
  \beta &= 0.3 rad.
\end{align*}
$$

During transition these three parameters vary linearly versus time. The other locomotion parameters stay the same during the simulation for the both legs : $L = 0.7m$ $H = 0.16m$ $T_x = T_y = 0$.

The pitch angle, vertical position of the gravity center and velocity of the robot are given in (fig. 6). We can clearly observe the two regimes and the transition period between them. The ground normal reaction forces are given in (fig. 7). In the first part, except at the instant of impact, the shapes of the curves are
similar as those recorded for a real human being. Furthermore, we can observe the step frequency, which increases versus time. Some snapshots of the simulation are shown on (fig. 8). Furthermore, the pitch angle decreases during the transition period and then stay stable with a more important oscillation frequency.

Figure 6: Smooth acceleration of the walking process ($t_1 = 5.4s$ and $t_2 = 9.4s$) by decreasing the cycle duration (from $T_c = 1.15s$ to $T_c = 0.9s$).

Figure 7: Normal forces exerted by the ground on the feet for the smooth acceleration (28 steps).

4.2 Quick acceleration

The second application is an acceleration of the walking process with a quick variation ($\frac{t}{2}$ s) of the locomotion parameters $T_c$, $\beta$ and $t_0$. All the locomotion parameters are the same as the previous simulation, but the time interval of transition is strongly reduced: $t_1 = 5.4s$ and $t_2 = 5.42s$. The pitch angle, vertical position of the gravity center and velocity of the robot are given in (fig. 9). The second walking regime is reached in a shorter time than the previous simulation but not instantaneously. The dynamics inertia of the system implies a longer transition as the desired one. Furthermore, the pitch angle frequency greatly increases at the transition period and then decreases. It should be noted that there is no feedback control and that the obtained cycles are intrinsically dynamically stable for our walking biped model.

4.3 Deceleration

In the third application, we want to decelerate the locomotion cycle by shortening the step length and to obtain a new gait. The varying parameters are the following: before transition ($t < t_1 = 5.4s$) : $L = 0.7m$, $H = 0.16m$, $\beta = 0.2rad$, $T_c = 0$; after transition ($t > t_2 = 9.4s$) : $L = 0.64m$, $H = 0.18m$, $\beta = 0.28rad$, $T_c = -0.2m$.

The others locomotion parameters stay the same during the simulation : $T_c = 1.15s$, $t_1^0 = 0$, $t_2^0 = 0$, $T_y = 0$, $x_{c1} = x_{c2} = -0.02m$, $y_{c1} = y_{c2} = -1.024m$.

The pitch angle, vertical position of the gravity center and velocity of the robot are given in (fig. 10). We can clearly observe the two regimes and the transition period between them. Some snapshots of the simulation for the new gait are shown on (fig. 11).
Figure 10: Deceleration of the walking process ($t_1 = 5.4s$ and $t_2 = 9.4s$) by decreasing the step length.

5 Conclusions and further developments

We have explained a method to generate the nominal regime of a walking robot based on statistical data concerning the joint angles during a steady walking of a human being. We have shown that a set of elementary spatial and temporal transformations of the locomotion parameters enables us to modify the typical walking cycle and obtain new gaits for the walking robot. Finally, some results about transitions were given showing the feasibility to accelerate or decelerate the locomotion cycle and to obtain new stable walking regimes.

The further developments are to establish, thanks to the elementary transformations, the transition between walking and running for an anthropomorphic walking robot. Since the mechanical structure and the gaits are completely defined by parameters, an optimization of both the locomotion and structural parameters is clearly possible.

References


Figure 11: Walking by decreasing step length and increasing step height.


